LV. *On the Change in the Obliquity of the Ecliptic, its Influence on the Climate of the Polar Regions and on the Level of the Sea.* By JAMES CROLL*.

IT is now pretty generally admitted by geologists that during the past geological history of our globe the northern hemisphere at least has passed through several glacial epochs. The existence of an ice-period during the Permian and Upper Miocene epochs, and perhaps during the Middle Eocene also, may now be regarded as established†. And I believe that there are few geologists who have given special attention to the matter who will not admit that we have evidence of the existence of ice-action during the Silurian, Old Red Sandstone, and Cretaceous periods. But what is most remarkable is the fact that during those very same periods we have evidence of the existence of a much warmer and more equable condition of climate than prevails at present. And we have also indisputable evidence that this warm and equable condition of climate was not confined to temperate regions, but extended beyond the arctic circle, even up to the highest latitudes that man has yet been able to reach. I shall state briefly a few of the facts to which I allude.

In the arctic regions encrinites, corals, and mollusca belonging to the Silurian period have been found in abundance‡, proving that a warm sea must have prevailed in those regions during at least a part of that age. It is well known that all over the arctic regions, extending to the most northern limit that has yet been attained, coal and carboniferous limestone have been found in abundance. And the existence of magnesian limestone in high latitudes seems to indicate that also during a part of the Permian period a warm sea must have reached to arctic regions.

We have evidence of a warm condition of climate in North Greenland during the Oolitic period. For example, in Prince Patrick's Island, at Wilkie Point, in lat. 76° 20′ N. and long. 117° 20′ W., oolitic rocks containing an ammonite (*Ammonites M'Clintocki*, Haughton) like the *Ammonites concavus* and other shells of the oolitic species were found by Captain M'Clintock§. In Katmai Bay near Behring's Straits the following oolitic fossils were found:—*Ammonites Wossnessenskii, Ammonites biplex, Belemnites paxillosus*, and *Unio liassinus*‖. Sir E. Belcher found

* Read before the Geological Society of Glasgow, April 18, 1867, and reprinted, by the permission of the Council, from their 'Transactions,' vol. ii. part 3.

† Lyell's 'Principles,' new edition, vol. i. chap. x. and xi.

‡ Quart. Journ. Geol. Soc. vol. xi. p. 510.

§ Journal of the Royal Dublin Society for February 1857.

‖ Quart. Journ. Geol. Soc. vol. xi. p. 519.

in Exmouth Island, lat. 77° 16′ N. and long. 96° W., at an elevation of 570 feet above the level of the sea, bones which were examined by Professor Owen and pronounced to be those of the *Ichthyosaurus**. Mr. Salter remarks that, at the time that these fossils were deposited, "a condition of climate something like that of our own shores was prevailing in latitudes not far short of 80° N."† And Mr. Jukes says that during the oolitic period, "in latitudes where now sea and land are bound in ice and snow throughout the year, there formerly flourished animals and plants similar to those living in our own province at the time. The questions thus raised," continues Mr. Jukes, "as to the climate of the globe when cephalopods and reptiles, such as we should expect to find only in warm or temperate seas, could live in such high latitudes, are not easy to answer"‡. When we come to examine the arctic flora of the Upper Miocene period, we find that North Greenland and the entire polar regions at that period enjoyed a condition of climate as warm as that of England at the present day. "We know," says Sir Charles Lyell, "that Greenland was not always covered with snow and ice; for when we examine the tertiary strata of Disco Island, we discover there a multitude of fossil plants which demonstrate that, like many other parts of the arctic regions, it formerly enjoyed a mild and genial climate§.

At the Meeting of the British Association held at Nottingham in August last, Professor Heer read a valuable paper on the Miocene Flora of North Greenland. In this paper some remarkable conclusions as to the probable temperature of Greenland during the Miocene period were given. Upwards of sixty different species brought from Atanekerdluk, a place on the Waigat opposite Disco, in lat. 70° N., have been examined by him.

A steep hill rises, he says, on the coast to a height of 1080 feet; and at this level the fossil plants are found. Large quantities of wood in a fossilized or carbonized condition lie about. Captain Inglefield observed one trunk thicker than a man's body, standing upright. The leaves, however, are the most important portion of the deposit, and give a most valuable insight into the nature of the vegetation which formed this primeval forest. In regard to those fossilized plants, he concludes that they cannot have been drifted from any great distance. They must have grown on the spot where they are found. And they prove without a doubt that North Greenland in the Miocene epoch had a

* The Last of the Arctic Voyages, by Captain Sir E. Belcher, vol. ii. p. 389, Appendix.

† Quart. Journ. Geol. Soc. vol. ii. p. 379.

‡ Manual of Geology, pp. 395, 493.

§ Antiquity of Man, second edition, p. 237.

climate much warmer than its present one. He states that it is quite impossible that the trees found at Atanekerdluk could ever have flourished there if the temperature were not far higher than at present. The difference must have been at least 29° F. Nor were the trees found here all at the extreme northern limit of their growth; for in the Miocene flora of Spitzbergen, lat. 78° N., we find the beech, plane, hazel-nut, and some other species identical with those from Greenland. And we may conclude, he thinks, that the firs and poplars which we meet at Atanekerdluk and Bell Sound, Spitzbergen, must have reached up to the North Pole if land existed there in the Tertiary period.

The *Sequoia Langsdorfii* is the most abundant of the trees of Atanekerdluk. The *Sequoia sempervirens* is its present representative. This tree has its extreme northern limit about lat. 53° N. For its existence it requires a summer temperature of 59° or 61°. Its fruit requires a temperature of 64° for ripening. The winter temperature must not fall below 34°; and that of the whole year must be at least 49°. The temperatures of Atanekerdluk during the time that the Miocene flora grew could not have been less than the above-mentioned. "These conclusions," says Professor Heer, "are only links in the grand chain of evidence obtained from the examination of the Miocene flora of the whole of Europe. They prove to us that we could not by any rearrangement of the relative positions of land and water produce for the northern hemisphere a climate which would explain the phenomena in a satisfactory manner. We must," he continues, "admit that we are face to face with a problem whose solution in all probability must be attempted, and, we doubt not, completed by the astronomer."

But, more singular still, at a period not anterior at least to that of the boulder-clay, a condition of climate prevailed in arctic regions much warmer than at present; for the remains of ancient forests have been found in places where at present nothing is to be seen but fields of snow and ice, and where the mean annual temperature scarcely rises above the zero of the Fahrenheit thermometer.

A trunk of a tree was discovered erect as it grew, by Captain Sir E. Belcher, on the 12th of September 1853, to the north of a narrow strait which opens into Wellington Sound, lat. 75° 32′ N., long. 92° W. The trunk was dug up and brought to England and examined by Dr. Hooker, who pronounced it to be a species of white spruce, *Pinus* (*Abies*) *alba**. The remains of an ancient forest were discovered by Captain M'Clure in Banks's Land, in lat. 74° 48′, extending along a range of hills varying from 350 to 500 feet above the sea, and from half a mile up-

* British Association Report for 1855, p. 101.

wards inland. He found a great accumulation of fossil trees, as well as fragments not fossilized lying over the whole extent of the land. "This remarkable phenomenon," says Captain M'Clure, "opens a vast field for conjecture; and the imagination becomes bewildered in trying to realize that period of the world's history when the absence of ice and a milder climate allowed forest trees to grow in a region where now the ground-willow and dwarf birch have to struggle for existence"*. Trunks of trees, some of them 3 feet in circumference, have been found in Prince Patrick's Island and Melville Island on the spot in which they grew. This place is perhaps at present the coldest spot in the northern hemisphere.

It is true that Sir Roderick Murchison† had at one time stated it to be his opinion that those vast quantities of wood found in arctic regions were drifted to their present positions at a period when those regions were submerged. This opinion is now, I believe, generally abandoned.

But the most remarkable circumstance, and certainly the most unaccountable of all upon the ordinary theories of change of climate during geological epochs, is the fact that we have not only had a succession of cold and warm periods, but cold and warm periods occurring in the same epoch. For example, during the Permian period we have not only evidence of a cold period when glaciers even in our own island reached to the sea-level‡, but we have also evidence, as we have seen, of a warm condition of climate extending during that epoch even to the arctic regions. And during the Upper Miocene period we find a cold climate prevailing and glaciers descending to the sea-level in the latitude of Italy§; while during another part of this period we know that the climate of the northern hemisphere was much warmer than at present, and Greenland free of ice and covered with a rich and luxuriant flora reaching up to perhaps the North Pole. Again, during the Pliocene period we have the well-known glacial epoch with the northern hemisphere to considerably low latitudes enveloped in one general capping of ice. And recent discoveries in arctic regions show that at that very period, or perhaps at a period somewhat later, huge forests flourished in North Greenland and the regions about Melville Island, where at present not a shrub can grow, and where nothing is to be seen but interminable fields of snow and ice.

How are these extraordinary changes of climate to be ac-

* Discovery of a North-west Passage, p. 208.

† Quart. Journ. Geol. Soc. vol. xi. p. 540.

‡ Ibid. p. 197.

§ Lyell's 'Principles,' vol. i. p. 207. Memoirs of Royal Academy of Sciences of Turin, second series, vol. xx.

counted for ? We require a cause not only of wide influence but of great intensity of action—in short, a cause which can at one time confer on Greenland, Siberia, and the entire arctic regions the mild climate of England or Madeira, and at another time bury England and nearly the entire temperate regions under perpetual snow and ice. And it must be able not only to do this, but to do it during the continuance of one epoch. There are few, I presume, who can seriously entertain the opinion that these changes can be satisfactorily accounted for without having reference to cosmical agency. When we view the matter from a purely physical and cosmical standpoint, we at once perceive that those changes, extreme as they no doubt appear, are in reality what we ought *à priori* to conclude must have occurred. I feel persuaded that those very conclusions to which geologists have been led regarding the changes of climate during past ages, would ultimately have all been arrived at through purely cosmical and physical considerations, even although geology as a science had not existed; they follow so obviously from theory.

It follows as a necessary conclusion, that when the excentricity of the earth's orbit reaches a very high value, the hemisphere which has its winter occurring in aphelion will be under a glacial condition, while the other hemisphere having its winter in perihelion will be enjoying a warm and equable climate. And the occurrence at times of cold and warm conditions of climate during the same geological epoch is in fact a necessary result of the precession of the equinoxes combined with that of excentricity.

There is still another cause which I feel convinced must to a very considerable extent have affected climate during past geological ages. I refer to the change in the Obliquity of the Ecliptic. This cause has long engaged the attention of geologists and physicists; and the conclusion generally arrived at, like that which had been arrived at in regard to excentricity, is that no great effect can be attributed to it. After giving special attention to the matter, I have been led to the very opposite conclusion. It is quite true, as has been urged, that the changes in the obliquity of the ecliptic cannot sensibly affect the climate of temperate regions. But it will produce a slight change on the climate of the tropical regions, and a very considerable effect on that of the polar regions, especially at the poles themselves. We shall now consider the matter briefly.

It was found by Laplace that the obliquity of the ecliptic will oscillate to the extent of $1° 22' 34''$ on each side of $23° 28'$, the obliquity in the year 1801. This change will but slightly affect the climate of the temperate regions, but it will exercise a very considerable influence on the climate of the polar regions. According to Mr. Meech*, if 365·24 thermal days represent the present total annual quantity of heat received at the equator from the sun, 151·59 thermal days will represent the quantity received at the poles. Adopting his method of calculation, it turns out that when the obliquity of the ecliptic is at its maximum, viz. at 24° 50′ 34″, the quantity received at the equator would be 363·51 thermal days, and at the poles 160·04 thermal days. The equator would therefore receive 1·73 thermal days less heat, and the poles 8·45 thermal days more heat than at present.

Annual Amount of Sun's Heat.

Amount in 1801. Obliquity 23° 28′.		Amount at maximum, 24° 50′ 34″.	Difference.
Latitude.	Thermal days.	Thermal days.	Thermal days.
0	365·24	363·51	−1·73
40	288·55	288·32	−0·23
70	173·04	179·14	+6·10
80	156·63	164·63	+8·00
90	151·59	160·04	+8·45

When the obliquity was at a maximum, the poles would therefore be receiving 19 rays for every 18 they are receiving at present. The poles would then be receiving nearly as much heat as latitude 76° is receiving at present.

The increase of obliquity would not sensibly affect the polar winter. It is true that it would slightly increase the breadth of the frigid zone; but the length of the winter at the poles would remain unaffected. After the sun disappears below the horizon his rays are completely cut off, so that a further descent of 1° 22′ 34″ would make no material difference whatever in the climate. In the temperate regions the sun's altitude at the winter solstice would be 1° 22′ 34″ less than at present. This would slightly increase the cold of winter in those regions. But the increase in the amount of heat received by the polar regions would materially affect the condition of the polar summer. What, then, is the rise of temperature at the poles which would result from the increase of 8·45 thermal days in the total amount received from the sun?

An increase of 8·45 thermal days, or $\frac{1}{18}$ of the total quantity received from the sun, according to the mode of calculation adopted in a former paper†, would produce, all other things being equal, a rise in the mean annual temperature equal to 14° or 15°.

* Smithsonian Contributions to Knowledge, vol. ix.
† Phil. Mag. for February 1867. Lyell's 'Principles,' vol. i. p. 294.

According to Professor Dove* there is a difference of 7°·6 between the mean annual temperature of lat. 76° and the pole,—the temperature of the former being 9°·8, and that of the latter 2°·2. Since it follows that when the obliquity of the ecliptic is at a maximum the poles would receive about as much heat per annum as lat. 76° receives at present, it may be supposed that the temperature of the poles at that period ought to be no higher than that of lat. 76° at the present time. A little consideration will, however, show that this by no means ought to be the case. Professor Dove's Tables represent correctly the mean annual temperature corresponding to every tenth degree of latitude from the equator to the pole. But it must be observed that the rate at which the temperature diminishes from the equator to the pole is not proportionate to the decrease in the total quantity of heat received from the sun as we pass from the equator to the pole. Were the mean annual temperature of the various latitudes proportionate to the amount of direct heat received, the equator would be much warmer than it actually is at present, and the poles much colder. The reason of this is perfectly obvious. There is a constant transference of *heat* from the equator to the poles, and of *cold* from the poles to the equator. The warm water of the equator is constantly flowing towards the poles, and the cold water at the poles is constantly flowing to the equator. The same is the case in regard to the aërial currents. Consequently a great portion of the direct heat of the sun goes, not to raise the temperature of the equator, but to heat the poles. And, on the other hand, the cold materials at the poles are transferred to the equator, and thus lower the temperature of that part of the globe to a great extent. The present difference of temperature between lat. 76° and the pole, determined according to the rate at which the temperature is found to diminish between the equator and the pole, amounts to only about 7° or 8°. But were there no mutual transference of warm and cold materials between the equatorial and polar regions, and the temperature of each latitude to depend solely upon the direct rays of the sun, the difference would far exceed that amount.

Now, when the obliquity of the ecliptic was at its superior limit and the poles receiving about $\frac{1}{18}$ more direct heat from the sun than at present, the increase of temperature due to this increase of heat would be far more than 7° or 8°. It would probably be nearly double that amount.

The enormous effect that ocean-currents have in equalizing the temperature of our globe, by diminishing the difference between the temperature of the equator and the poles, has never

* Distribution of Heat on the Surface of the Globe, p. 14.

been duly estimated. This will be seen if we merely consider for a moment the effect produced by one current alone, viz. the Gulf-stream. The total quantity of water conveyed by this stream is probably equal to that of a stream 50 miles broad and 1000 feet deep, flowing at the rate of 4 miles an hour. And the mean temperature of the entire mass of moving water is not under 65° at the moment of leaving the Gulf*. I think we are warranted to conclude that the Gulf-stream, before it returns from its northern journey, is on an average cooled down at least 25°. Each cubic foot of water, therefore, in this case carries from the tropics upwards of 1500 units of heat, or 1,158,000 foot-pounds. According to the above estimate of the size and velocity of the stream, 5,575,680,000,000 cubic feet of water are conveyed from the Gulf per hour, or 133,816,320,000,000 cubic feet daily†. Consequently the total quantity of heat transferred from the equatorial regions per day by the stream amounts to 154,959,300,000,000,000,000 foot-pounds. From observations made by Sir John Herschel and by M. Pouillet on the direct heat of the sun, it is found that, were no heat absorbed by the atmosphere, about 83 foot-pounds per second would fall upon a square foot of surface placed at right angles to the sun's rays‡. Mr. Meech estimates that the quantity of heat cut off by the atmosphere is equal to about 22 per cent. of the total amount received from the sun. M. Pouillet estimates the loss at 24 per cent. Taking the former estimate, 64·74 foot-pounds per second will therefore be the quantity of heat falling on a square foot of the earth's surface when the sun is in the zenith. And were the sun to remain stationary in the zenith for twelve hours, 2,796,768 foot-pounds would fall upon the surface.

It can be shown that the total amount of heat received upon a unit-surface on the equator during the twelve hours from sunrise till sunset at the time of the equinoxes is to the total amount which would be received upon that surface, were the sun to remain in the zenith during those twelve hours, as the diameter of a circle to half its circumference, or as 1 to 1·5708. It follows, therefore, that a square foot of surface on the equator receives

* Phil. Mag. for February 1867, p. 127.

† Captain Maury considers the Gulf-stream equal to a stream 32 miles broad and 1200 feet deep, flowing at the rate of five knots (30,415 feet) an hour (Physical Geography of the Sea, § 24). This gives 6,166,700,000,000 cubic feet per hour as the quantity of water conveyed by the stream. Sir John Herschel's estimate is still greater. He considers it equal to a stream 30 miles broad and 2200 feet deep, flowing at the rate of four miles an hour (Physical Geography, § 54). This makes the quantity 7,359,900,000,000 cubic feet per hour. Sir John estimates the temperature at 86° F.

‡ Trans. of Royal Soc. of Edin. vol. xxi. p. 57. Phil. Mag. S. 4. vol. ix. p. 36.

from the sun at the time of the equinoxes 1,780,474 foot-pounds daily, and a square mile 49,636,750,000,000 foot-pounds daily. But this amounts to only $\frac{1}{3,121,870}$ part of the quantity of heat daily conveyed from the tropics by the Gulf-stream. In other words, the Gulf-stream conveys as much heat as is received from the sun by 3,121,870 square miles of surface at the equator. The amount thus conveyed is equal to all the heat which falls within 63 miles on each side of the equator. According to calculations made by Mr. Meech, the annual quantity of heat received by a unit-surface on the frigid zone, taking the mean of the whole zone, is $\frac{5 \cdot 45}{12}$ of that received at the equator. Consequently the quantity of heat conveyed by the Gulf-stream in one year is equal to the heat which falls on an average on 6,873,800 square miles of the arctic regions. The frigid zone or arctic regions contain 8,130,000 square miles. There is actually, therefore, nearly as much heat transferred from the tropical regions by the Gulf-stream as is received from the sun by the entire arctic regions, the quantity conveyed by the stream to that received from the sun by those regions being as 15 to 18.

But we have been assuming in our calculations that the percentage of heat absorbed by the atmosphere is no greater in polar regions than it is at the equator, which is not the case. If we make due allowance for the extra amount absorbed in polar regions in consequence of the obliqueness of the sun's rays, the total quantity of heat conveyed by the Gulf-stream will probably nearly equal the amount received from the sun by the entire arctic regions.

It may be stated, however, that the extra amount of heat absorbed in polar regions in consequence of the oblique direction in which the rays pass through the atmosphere is not nearly so great as is generally represented. It is commonly assumed that the quantity of heat absorbed by the atmosphere is proportionate to the number of aërial particles that the rays have to encounter before reaching the surface of the earth. It is stated as a general rule that, if the tracks of the rays follow an arithmetical progression, the diminished force with which the rays reach the ground will form a decreasing geometrical progression. But recent discoveries in regard to the absorption of radiant heat by gases and vapours prove that Tables computed on this principle must be wholly erroneous. The researches of Tyndall and Melloni show that, when a ray passes through any substance, the absorption is rapid at first, but the ray is soon "sifted," as it is called, and it then passes onwards with but little further obstruction.

In the article on climate in the *Encyclopædia Britannica*, where the calculations are made upon the foregoing principles, it is stated that the annual intensity of the sun's heat at the equator to that at the poles is as 115 to 14, or as 8 to 1 nearly. If the quantity of heat received from the sun by polar regions were actually as small as the writer of this article states, it would follow that the quantity conveyed by the Gulf-stream from the equatorial regions is equal to about three times all that falls within the arctic circle.

It is true that a very considerable part of the heat thus transferred by currents from the equator never reaches the arctic regions. But it nevertheless goes to lower the temperature of the equator and raise the temperature of the higher latitudes. Let it be remembered that our calculations refer only to one single current, the Gulf-stream. If we took into account all the currents which flow from the equatorial regions of our globe, we should find that their influence in reducing the difference between the temperature of equatorial and polar regions is enormous.

A similar function is also performed by aërial currents. The equatorial regions are cooled to a great extent by an aërial current continually flowing from the polar regions to the equator. And the polar regions at least are warmed by an upper current flowing from the torrid and temperate regions towards the poles. The equator is also cooled to a very great extent by *convection.* The hot surface of the ground at the equator transfers its heat to the air, and an ascending current is generated which carries the heat upwards and dissipates it into stellar space.

It is very probable that were it not for the combined effect of all those causes, the difference of temperature between the equator and the poles would amount to at least 200° instead of about 80°, as at present. Did the earth possess neither an atmosphere nor seas, and were the temperature of each place to depend entirely on the direct heat of the sun, the difference of mean annual temperature between the equator and the poles, according to theory, ought to amount to about 200°.

We may therefore conclude that when the obliquity of the ecliptic was at a maximum, and the pole receiving $\frac{1}{18}$ more heat than at present, the temperature of the poles ought to have been about 14° or 15° warmer than at the present day, *provided, of course, that this extra heat was employed wholly in raising the temperature.* Were the polar regions free from snow and ice, the greater portion of the extra heat would go to raise the temperarature. But as those regions are covered with snow and ice, the extra heat would have no effect in raising the temperature, it would simply melt the snow and ice. The ice-covered surface upon which the rays fell could never rise above 32°. At the period under consideration the total annual quantity of ice melted at the poles would be $\frac{1}{18}$ more than at present.

The general effect which the change in the obliquity of the ecliptic would have upon the climate of the polar regions when combined with the effects resulting from the excentricity of the earth's orbit would be this:—When the excentricity was at a very high value, the hemisphere whose winter occurred in aphelion (for physical reasons which have already been discussed at considerable length*) would be under a condition of glaciation, while the other hemisphere, having its winter in perihelion, would be enjoying a warm and equable climate. When the obliquity of the ecliptic was at a maximum, and $\frac{1}{18}$ more heat falling at the pole than at present, the effect would be to modify to a great extent the rigour of the glaciation in the polar zone of the hemisphere under a glacial condition, and, on the other hand, to produce a more rapid melting of the ice on the other hemisphere enjoying the equable climate. The effects of excentricity and obliquity thus combined would probably completely remove the polar ice-cap from off the latter hemisphere, and forest-trees might then grow at the pole. Again, when the obliquity was at its minimum condition and less heat reaching the poles than at present, the glaciation of the former hemisphere would be increased and the warmth of the latter diminished.

One very remarkable effect which seems to result indirectly from a variation of the obliquity, under certain conditions, is an influence on the level of the sea. As this probably may have had something to do with those recent changes of sea-level which the history of the submarine forests and raised beaches have made us all so familiar with, it may be of interest to enter at some length into this part of the subject.

It appears almost certain that, at the time when the northern winter solstice was in aphelion last, a rise of the sea on the northern hemisphere to a considerable number of feet must have taken place from the combined effect of excentricity and obliquity. About 11,700 years ago the northern winter solstice was in the aphelion. The excentricity at that time was ·0187, being somewhat greater than it is now; but the winters occurring in aphelion instead of perihelion, as at present, they would on this account be probably 10° or 15° colder than they are at the present day. It is probable also, for reasons formerly stated†, that the Gulf-stream at that time would be considerably less than at present. This would tend to lower the temperature to a still greater extent. As snow must have fallen during winter instead of rain, this no doubt would have produced a slight increase in the quantity of ice on the northern hemisphere, had no other cause come

* Phil. Mag. for August 1864 and February 1867. † Ibid.

into operation. But the condition of things, we have every reason to believe, must have been affected by the greater obliquity of the ecliptic at that period. We have no formula from which we can determine with perfect accuracy the extent of the obliquity at a period so remote as the one under consideration. If we adopt the formula given by Struve and Peters, we have the obliquity at a maximum about the time that the solstice-point was in aphelion. The formula given by Leverrier places the maximum somewhat later. At all events we cannot be far from the truth in assuming that at the time the northern winter solstice was in aphelion the obliquity of the ecliptic was about a maximum, and that since then it has been gradually diminishing. It is evident, then, that the annual amount of heat received by the arctic regions, and especially about the pole, would be considerably greater than at present. And as the heat received on those regions is chiefly employed in melting the ice, it is probable that the extra amount of ice which would then be melted in the arctic regions would prevent that slight increase of ice which would otherwise result in consequence of the winter occurring in aphelion. The winters at that period would be colder than they are at present; but the total quantity of ice on the northern hemisphere would not probably be greater. Let us now turn to the southern hemisphere. As the southern winter would then occur in perihelion, it would tend to produce a slight decrease in the quantity of ice on the southern hemisphere. But on this hemisphere the effects of excentricity would not, as on the northern hemisphere, be compensated by those of obliquity; for both causes would here tend to produce the same effect, namely a melting of the ice in the antarctic regions.

It is probable that at this time the quantity of warm water flowing from the equatorial regions into the Southern Ocean would be much greater than at present. This would tend to raise the temperature of the air of the antarctic regions, and thus assist in melting the ice. These causes, combined with the great increase of heat resulting from the change of obliquity, would tend to diminish to a considerable extent the quantity of ice on the southern hemisphere. I think we may assume that the slight increase of excentricity at that period, the occurrence of the southern winter in perihelion, and the extra quantity of warm water flowing from the equatorial to the antarctic regions would produce an effect on the south polar ice-cap equal to that produced by the increase in the obliquity of the ecliptic. It would therefore follow that for every eighteen pounds of ice melted annually at present at the south pole twenty pounds would then be melted.

Let us now consider the effect that this condition of things would have upon the level of the sea. It would evidently tend to produce an elevation of the sea-level on the northern hemisphere, in two ways. 1st. The addition to the sea occasioned by the melting of the ice from off the antarctic land would tend to raise the general level of the sea. 2ndly. The removal of the ice would also tend to shift the earth's centre of gravity to the north of its present position; and as the sea must shift along with the centre, a rise of the sea on the northern hemisphere would necessarily take place.

The question naturally suggests itself, Might not the last rise of the sea, relative to the land, have resulted from this cause? We know that during the period of the 25-feet beach, the time when the estuarine mud which now forms the rich soil of the Carses of the Forth and Tay was deposited, the sea, in relation to the land, stood at least 20 or 30 feet higher than at present. But immediately prior to this period we have the age of the submarine forests and peat-beds, when the sea relative to the land stood lower than it does at present. We know also that these changes of level were not a mere local affair. There seems every reason to believe that our Carse clay, as Mr. Fisher states, is the equivalent of the marine mud with *Scrobicularia* which covers the submarine forests of England*. And, on the other hand, those submarine forests are not confined to one locality. "They may be traced," says Mr. Jamieson, "round the whole of Britain and Ireland, from Orkney to Cornwall, from Mayo to the shores of Fife, and even, it would seem, along a great part of the western seaboard of Europe, as if they bore witness to a period of wide-spread elevation, when Ireland and Britain, with all its numerous islands, formed one mass of dry land, united to the continent, and stretching out into the Atlantic"†. "These submarine forests," remarks De la Beche also, "are to be found under the same general condition from the shores of Scandinavia to those of Spain and Portugal, and around the British islands"‡. Those buried forests are not confined to Europe, but are found in the valley of the Mississippi and in Nova Scotia and other parts of North America. And, again, the strata which underlie those forests and peat-beds bear witness to the fact of a previous elevation of

* Quart. Journ. Geol. Soc. June 1866, p. 564.

† Ibid. vol. xxi. p. 186.

‡ Geological Observer, p. 446. See also Mr. James Geikie's valuable Memoir "On the Buried Forests and Peat Mosses of Scotland," Trans. of the Society of Edinburgh, vol. xxiv.; and Chambers's 'Ancient Sea-Margins.'

the sea-level. In short, we have evidence of a number of oscillations of sea-level during post-Tertiary times*.

Had there been but only one rise of the land relative to the sea-level, or one depression, it might quite reasonably have been attributed to an upheaval or a sinking of the ground, occasioned by some volcanic, chemical, or other agency. But certainly those repeated oscillations of sea-level, extending as they do over so wide an area, look more like a rising and sinking of the sea than of the land. But be this as it may, since it is now established, I presume, beyond controversy that the old notion that the general level of the sea remains permanent, and that the changes must be all attributed to the land, is wholly incorrect, and that the sea, as well as the land, is subject to changes of level, it is certainly quite a legitimate subject of inquiry to consider whether the last elevation of the sea-level relatively to the land may not have resulted from the rising of the sea rather than from the sinking of the land—in short, whether it may not be attributed to the cause we are now considering. The fact that those raised beaches and terraces are found at so many different heights, and also so discontinuous, along our coasts, might be urged as an objection to the opinion that they were due to changes in the level of the sea itself. Space will not permit of us entering upon the discussion of this point at present; but it may be stated that this objection is more apparent than real. It by no means follows that beaches of the same age must be at the same level. This has been shown very clearly by Mr. W. Pengelly in a paper on "Raised Beaches," read before the British Association at Nottingham last year.

We have, I presume, evidence amounting to almost absolute certainty that 11,700 years ago the general sea-level on the northern hemisphere must have been higher than at present. And in order to determine the question of the 25-feet beach, we have merely to consider whether a rise to something like this extent probably took place at the period in question. We have at present no possible means of knowing with certainty the extent of the rise which must have taken place at that period; for we have no means of ascertaining what was the quantity of ice which must have then been melted off the antarctic regions. But we have the means of making a very rough estimate, which, at least, may enable us to determine whether a rise of some 20 or 30 feet may not possibly have taken place. The principal difficulty which meets us is the uncertainty that still prevails regarding the amount of land within the antarctic

* See Lyell's 'Antiquity of Man,' second edition, p. 282; 'Elements,' sixth edition, p. 162.

circle. As to the existence of an antarctic continent there are, I presume, few well informed on the subject who entertain a doubt, whatever their views may be regarding its extent. If we examine the matter carefully, we shall find that we have every reason to believe that this continent extends, on an average, down to about latitude 70° S. It is only at a very few points that voyagers have been able to penetrate the "pack" and reach to anything like high latitudes. But it has been found that wherever a high latitude was reached *land was generally seen.* For example, lat. 66° 30′ was reached by M. D'Urville in long. 140°, and Adelie Land was found. Lat. 65° was reached by Balleny in long. 120° E., and Sabrina Land was found. Lat. 67° was reached by Biscoe, in long. 60° and 50° E., and Kemp Land and Enderby Land were found. Lat. 63° and 68° were reached by Biscoe, between long. 60° and 70° W., and Graham Land was found. In the year 1774 Capt. Cook reached lat. 71° 10′, in long. 106° 54′ W., and saw to the south a very extensive field of ice. And he at the time expressed it as his opinion that the ice covered a continent reaching to the south pole*. About long. 180° Victoria Land was discovered, and the great ice-barrier attached to it traced by Sir James Ross for several hundreds of miles. A perpendicular wall of ice, similar to that described by Sir James Ross, had been previously traced by the United States Exploring Expedition under Capt. Wilkes, from near the longitude of Victoria Land, in lat. 65°, along in the direction of Enderby Land, for a distance of upwards of 1500 miles. Along this extended coast-line nothing in the shape of an open strait, or of a current in a northerly direction, was found, which seems to show, as Capt. Wilkes remarks, that the ice-barrier must be attached to an unbroken tract of land†.

It is extremely probable that these various tracts of land are but different parts of one great continent extending to the south pole. If they are islands, it is singular that none of them have been circumnavigated. Besides, were they merely islands, it is difficult to conceive how there should be such prodigious masses of icebergs on the southern hemisphere, extending down in all directions to such comparatively low latitudes. If those various tracts of land are parts of one great continent, then this continent will on an average extend down to lat. 70°. I may mention that Sir Charles Lyell, who has given much attention to this subject, has, in his late edition of the 'Principles,' represented the antarctic continent as extending down to about lat. 70°.

* Cook's Voyages, vol. ii. p. 86.

† Narrative of United States Exploring Expedition: Whittaker and Co., London.

One thing, however, is certain, that this continent, whatever its extent may be, is covered with a vast sheet of ice. This sheet is known to be upwards of 1000 feet thick at its edges. And there are reasons of a mechanical nature (which will come under our notice perhaps on a future occasion) which prove that the thickness of the sheet in the centre of this continent must be enormous.

Assuming, then, that the southern ice-cap extends on an average down to lat. 70°, it will therefore be equal to $\frac{1}{33\cdot163}$ of the entire surface of the globe. The proportion of land to that of water, taking into account the antarctic continent, according to Mr. Saunders*, is as 526 to 1272. The southern ice-cap will therefore be equal to $\frac{1}{23\cdot46}$ of the area covered by water. The density of ice to that of water being taken at ·92 to 1, it follows that 25 feet 6 inches of ice melted from off the face of the antarctic continent would raise the level of the ocean 1 foot. If 470 feet were melted off—and this is by no means an extravagant supposition, when we reflect, that for every 18 pounds of ice presently melted an additional pound or two pounds, or perhaps more, would then be melted, and that for many ages in succession—the water thus produced from the melted ice would raise the level of the sea 18 feet 5 inches. The removal of the 470 feet of solid ice (which must be but a very small fraction of the total quantity of ice lying upon the antarctic continent) would shift the earth's centre of gravity about 7 feet to the north of its present position. The shifting of the centre of gravity would cause the sea to sink on the southern hemisphere and rise on the northern†. And the quantity of water thus transferred from the southern hemisphere to the northern would carry the centre of gravity abont 1 foot further, and thus give a total displacement of the centre to the extent of about 8 feet. The sea would therefore rise about 8 feet at the North Pole, and in the latitude of Glasgow about 6 feet 7 inches. This, added to the rise of 18 feet 5 inches, occasioned by the melting of the ice, would give 25 feet as the total rise in the latitude of Scotland 11,700 years ago.

Each square foot of surface at the poles 11,700 years ago would be receiving 18,223,100 foot-pounds more of heat annually than at present. If we deduct 22 per cent. as the amount absorbed in passing through the atmosphere, we have 14,214,000 foot-pounds. This would be sufficient to melt 2·26 feet of ice. But if 50 per cent. were cut off instead of 22 per cent., 1·45 cubic foot would be melted. In this case the 470

* Lyell's 'Principles,' vol. i. p. 257.
† Phil. Mag. for April 1866.

feet of ice would be melted, independently of the effects of excentricity, in about 320 years. And supposing that only one-fourth part of the extra heat reached the ground, 470 feet of ice would be removed in about 640 years.

As to the exact time that the obliquity was at a maximum previous to that of 11,700 years ago our uncertainty is still greater. If we are permitted to assume that the ecliptic passes from its maximum to its minimum state and back to its maximum again with anything like uniformity, at the rate assigned by Leverrier and others, the obliquity would not be far from a maximum about 60,000 years ago. Taking the rate of precession at $50''\cdot21129$, and assuming it to be uniform (which it probably is not), the winter solstice would have been in aphelion about 61,300 years ago*. In short, it seems not at all improbable that at the time the solstice-point was in the aphelion, the obliquity of the ecliptic would be not far from its maximum state. But at that time the value of the excentricity was 0·023, instead of 0·0187, its value at the last period. Consequently the rise of the sea would probably be somewhat greater than it was 11,700 years ago. Might not this be the period of the 40-feet beach? In this case 11,000 or 12,000 years would be the age of the 25-feet beach, and 60,000 years the age of the 40-feet beach.

About 22,000 years ago the winter solstice was in perihelion; and as the excentricity was then somewhat greater than it is at present, the winters would be a little warmer and the climate more equable than it is at the present day. This perhaps might be the period of the submarine forests and lower peat-beds which underlie the Carse clays, Scrobicularia mud, and other deposits belonging to the age of the 25-feet beach. At any rate, it is perfectly certain that a condition of climate at this period prevailed exceedingly favourable to the growth of peat. It follows also that at this time, owing to a greater accumulation of ice on the southern hemisphere, the sea-level would be a few feet lower than at present, and that forests and peat may have then grown on places which are now under the sea-level.

* In order to determine the position of the solstice-point in relation to the aphelion, it will not do to assume, as is commonly done, that the point makes a revolution from aphelion to aphelion in any regular given period, such as 21,000 years; for it is perfectly evident that, owing to the great irregularity in the motion of the aphelion, no two revolutions will probably be performed in the same length of period. For example, the winter solstice was in aphelion about the following dates: 11,700, 33,300, and 61,300 years ago. Here are two consecutive revolutions, the one performed in 21,600 years, and the other in 28,000 years, the difference in the length of the two periods amounting to no fewer than 6400 years.

For a few thousand years before and after 11,700 years ago, when the winter solstice was evidently not far from aphelion, and the sea standing considerably above its present level, would probably, as we have already stated, be the time when the Carse clays, the stratified sand and mud which form our Glasgow Green, and other recent deposits lying above the present level of the river, were formed. And it is also a singular fact that the condition of things at this period must have been exceedingly favourable to the formation of such estuarine deposits; for at this time the winter temperature of our island, as has been already shown, would be a few degrees below the freezing-point, and consequently during that season snow would fall instead of rain. The melting of the winter's accumulation of snow on the approach of summer would necessarily produce great floods, similar to what occur in the northern parts of Asia and America at the present day from this very same cause. The loose upper soil would be carried down by those floods and deposited in the estuaries of our rivers.

The foregoing is a rough and imperfect sketch of the history of the climate and physical conditions of our globe for the past 60,000 years, so far as physical and cosmical considerations seem to afford us information on the subject; and its striking agreement with that derived from geological sources is an additional evidence in favour of the opinion that geological and cosmical phenomena are physically related by a bond of causation.

We have assumed 1° 22′ 34″ greater than the present obliquity as the maximum. But this limit assigned by Laplace does not hold true for all time past *. And Sir John Herschel thinks that when millions of years are taken into account, the obliquity may even amount to 3° or 4° on each side of a mean state †.

Suppose, then, that at some past geological epoch the increase of the obliquity reached 4°, the equator would then be receiving 5·64 thermal days less heat than at present, and the poles upwards of 24 thermal days more heat than at present. The total quantity of heat received by the poles would therefore be nearly $\frac{1}{6}$ more than at the present day. For every 6 pounds of ice now melted at the pole, 7 pounds would then be melted. Were this condition of things to occur when the excentricity was near its superior limit, the hemisphere which had its winter occurring in perihelion at the time, would in all probability be completely deprived of its ice-cap, and forest-trees might then grow at the pole.

* *Mécanique Céleste,* vol. ii. p. 857, note, Bowditch's translation.
† Lyell's 'Principles,' new edition, vol. i. p. 282.

Were the obliquity increased by 4°, each square foot of surface at the poles would receive annually 51,802,000 foot-pounds more heat than at present, assuming that all the extra heat reached the ground. But suppose one half of the rays to be absorbed in passing through the atmosphere, 25,901,000 foot-pounds per square foot would then represent the extra heat received by the poles. This would be sufficient to melt 4·12 cubic feet of ice. The additional heat resulting from the increase of 4° in the obliquity of the ecliptic would consequently melt a sheet of ice one mile in thickness in about 1300 years. A mile of ice removed from off the antarctic continent (and this is not an extravagant quantity, as we hope to be able to show on a future occasion) would raise the general level of the sea 206 feet. And to this must be added the rise resulting from the displacement of the earth's centre of gravity, which in the latitude of Scotland would amount to about 74 feet. This would give 280 feet as the total rise resulting from the melting of one mile of ice from off the face of the continent. It is quite conceivable that an enormous amount of ice might be melted on the southern hemisphere, while the quantity on the northern hemisphere remained undiminished. For if this extreme condition of obliquity were to occur at the time that the excentricity was at a higher value (a very likely event), and the winter solstice in aphelion, the effects of excentricity would neutralize those of obliquity on the northern hemisphere, and consequently the quantity of ice on this hemisphere might not be diminished. But on the southern hemisphere the obliquity and the excentricity would conspire to produce the same effect, viz. a melting of the ice; and hence the effects that we have been considering, which would result from the removal of the ice, would necessarily take place to a greater or less extent.

The enormous effect that an extreme condition of excentricity must have in raising the temperature of the polar regions of the hemisphere which has its winter occurring in perihelion at the time, has been shown on a former occasion *. When such a state of things happens at the time when the obliquity is also at its superior limit, the combined effect of both causes would in all probability completely remove the polar ice-cap, and a submergence of the land on the opposite hemisphere to the extent of hundreds of feet would be the inevitable result.

It has been a matter of surprise how forest-trees could have flourished near the poles, seeing that they must have been deprived of the sun's rays for several months in the year. But it must be observed that it is not necessary that trees should *grow* during winter, when deprived of the sun's rays. All that

* Phil. Mag. for February 1867.

is required is, that they should not die during that season. But all botanists know perfectly well that a forest-tree would not perish simply in consequence of being deprived of the direct rays of the sun for a few months, provided that it enjoyed a sufficient amount of heat. We are, however, not even warranted to conclude that the absence of the sun would prevent growth. It would affect the colour of the leaves; but it is doubtful whether it would seriously affect the growth of the tree. From a paper read before the French Academy, April 9, 1866, by M. P. Duchartre, it would appear that some plants grow much faster during the night, when deprived of the sun's rays, than during daylight.

LVI. *On the Problem on Attractions, in the Philosophical Magazine, page* 332. *By* Archdeacon Pratt, *M.A., F.R.S.*

To the Editors of the Philosophical Magazine and Journal.

Gentlemen,

THE latter part of the solution I sent you, March 7, 1867, may also be done as follows:—

I have shown that the condition

$$\int_{-1}^{1}\int_{0}^{2\pi} \mathrm{P}_i \frac{u^{i+3}}{i+3}\left\{\rho^{(0)} - \frac{\rho^{(1)}}{i+4} + \&\text{c.}\right\} d\mu\, d\omega = 0$$

must be satisfied, i being any whole number, and u, $\rho^{(0)}$, $\rho^{(1)}\dots$ functions of μ and ω, and independent of i; and P_i Laplace's coefficient of the ith order, the same function of μ' and ω' (the coordinates of the external attracted point) as of μ and ω. Put the whole expression under the signs of integration, except P_i, equal to $\phi\,(\mu, \omega) = \phi_0 + \phi_1 + \dots$ a series of Laplace's functions. Then by a property of those functions*,

$$\int_{-1}^{1}\int_{0}^{2\pi} \mathrm{P}_i \phi(\mu, \omega)\, d\mu\, d\omega = \frac{4\pi}{2i+1}\phi'_i,$$

where ϕ'_i is the same function of μ' and ω' that ϕ_i is of μ and ω. But this expression equals zero by the condition. Hence $\phi'_i = 0$ and also $\phi_i = 0$ for all integral values of i;

$$\therefore \frac{u^{i+3}}{i+3}\left\{\rho^{(0)} - \frac{\rho^{(1)}}{i+4} + \dots\right\} = \phi_0,$$

where ϕ_0 is independent of μ and ω, and therefore of u, and a function of i only. This must be true, however large i is. Now $\rho^{(0)}$, the density at the surface, must always be some finite quantity. Consequently, as the condition now stands, when i is taken

* See my 'Figure of the Earth,' 3rd. edit. p. 29.