LI. *On Maps of the World.* *By* George Darwin, *M.A., Fellow of Trinity College, Cambridge*.*

[With a Plate.]

THE ordinary stereographic projection of the world in two hemispheres is utterly worthless as giving a true impression of the whole; for the linear scale at the margins of the circles is twice that at their centres. Its only merit is that there is no angular distortion. Mercator's projection gives a still more fallacious impression, except as regards the equatorial regions.

It appears to me therefore that there is a want, in the school-room and lecture-room, of some map which shall give a more truthful representation of the globe than the above, and which yet shall not be so expensive and cumbrous as a globe.

A gnomonic projection on to the faces of a regular icosahedron is but very slightly distorted, although a slight amount of angular distortion is here introduced. I have been told that at the recent Geographical Congress at Paris, some such projections as this were exhibited, and that they were of old date. Mr. Proctor has also made star-maps by projection on to the faces of a regular dodecahedron; but in 1872, when the idea occurred to me of using this projection, I was not aware of the fact.

If the icosahedral projection be developed and arranged as a band of ten triangles round the equator, with saw-like edges of five triangles in the north and five in the south, a very fair representation of the globe is given. And the interstices between the teeth of the saws may be arranged so as not to damage the continents very severely.

In this map the meridians are straight lines, but are broken in direction at the junction of two triangles. The parallels of latitude become ellipses, which may be easily laid out by aid of a property of conic sections; viz. if a circular cone be placed with its vertex at the centre of a sphere, and a section made by a tangent plane to the sphere, the radius of curvature at the vertices of this conic section is constant for all tangent planes, and varies as the tangent of the semiangle of the cone.

Now in our map the ellipses are represented with sufficient accuracy by the circles of curvature at their vertices; and the radii of these circles may be taken direct with the compasses from a sector, as the cotangents of the corresponding latitudes.

Besides a map of this kind, I have also constructed a portable quasi-globe with this method of projection. The faces of the

* Communicated by the Author.

icosahedron are made to hinge together, so that the whole can be packed flat in the form of a half-hexagon. Such a globe was exhibited at the British-Association Meeting at Bradford. When mounted, the icosahedron circumscribed a sphere of 25 inches diameter. This form of globe might doubtless be constructed much cheaper than a truly spherical one, because the framework would be ordinary carpentry, and the twenty map-sheets might be printed flat like ordinary maps.

In 1872 I showed the above described maps and globe to General Strachey; and he suggested that by cutting down the iscosahedron in some way, a still more satisfactory projection might be attained. It then occurred to us that by truncating the solid angles of the icosahedron, a solid figure of 32 faces would be obtained, viz. 20 hexagons and 12 pentagons.

If the truncation be carried on by slices until the truncating planes touch the sphere enclosed in the icosahedron, these hexagons are not regular, but have two sets of three sides equal to one another; a long side is always opposite to a short side. If unity is the radius of the sphere, the long sides and short sides are respectively ·4913 and ·3401. The pentagons are always regular; and at this particular degree of truncation the side of the pentagon is ·4913, and a pentagon is therefore always contiguous to the long side of a hexagon; whilst hexagons are always contiguous along their short sides*.

This projection was utilized by having a sort of umbrella-like stand, with a pentagonal face in the middle, surrounded by five hexagons; or else with a hexagon in the middle, surrounded by three pentagons and by three hexagons. The maps were drawn on 32 separate sheets; and the sheets required to represent any part of the world were mounted on the umbrella.

By these means about one fifth of the globe is shown at once; and thus the equivalent of a very large globe might be used in a room of ordinary size. The sheets may also be conveniently kept, since they are all flat, and will lie one on another.

The figures 1, 2, 3, 4 (Plate I.) show the forms of the various map-sheets, together with the figures required for laying out the meridians and parallels of latitude. Besides those kinds shown in the figures, there are two pentagons which close in the two poles; but it is so easy to lay them out, that it does not seem

* This leads me to observe that if the angles of any one of the regular solids be truncated in this way, another one is ultimately produced. The 20-hedron and 12-hedron, the 8-hedron and cube, and the tetrahedron and tetrahedron are thus correlated. This property is of course due to the fact that the polar reciprocal of any regular solid is itself a regular solid. It is curious to observe the transitional forms as the slices are cut off the angles.

worth while to give a figure. The meridians on the equatorial faces converge so little that it is more convenient to set them out by finding two points through which they pass. The broken lines in the figures are merely constructional.

In order that the meridians and lines of latitude may fall symmetrically on each face, it is better to set them every 9° or 6°, instead of every 10 as is usually done. For the whole globe, there are required 10 equatorial hexagons, 10 equatorial pentagons (5 in N. and 5 in S.), and 2 polar pentagons.

This 32-faced figure is a very close approximation to the globe.

The Murchison Fund of the Geographical Society (£40) has been granted for carrying this scheme out practically; and a Committee has been appointed, of which General Strachey and Mr. Francis Galton are members. The scale is large, the polyhedron being designed to circumscribe a sphere of 10 feet diameter. The various sheets of the map are stretched on light wooden frames; and they can be hasped on to a kind of umbrella, of which the handle is held horizontal. It is expected that it will be finished shortly; and it will, I believe, be placed in the rooms of the Society.

Another somewhat similar plan has occurred to me, and seems to me preferable, at any rate for somewhat smaller globes than the one above referred to.

Suppose A B C to be one face of a regular icosahedron *inscribed* in a sphere (see fig. 5), and that we bisect the arcs of great circles subtended by the sides A B, B C, C A respectively in D, E, F. Then pass a plane through D E F, and three others through A E F, B D F, C D E respectively. The face A B C may be replaced by the equilateral triangle D E F and the three isosceles triangles A E F, B D F, C D E. If this be done with every face of the icosahedron, we have a solid figure of 80 faces—20 equilateral triangles, and 60 isosceles (but nearly equilateral) triangles—inscribed in the sphere. If we project the globe on to this surface, with the vertex of projection at the centre, we obtain an excellent approximation to the true globe.

Now this plan would be very complicated if it were necessary to have 80 different map-sheets. Fortunately, however, the form of the triangles makes it advantageous to have four sheets united together, viz. the equilateral triangle and the three isosceles ones which have replaced the face of the original icosahedron.

Fig. 6 represents one of these sets of four sheets when spread out flat. These four sheets may be printed from a single plate, and may be pasted on to quasi-triangles, such as A B C (fig. 6), which are hinged or creased along the lines D E, E F, F D.

If the scale on which it is carried out is sufficiently small to permit of the faces being made of cardboard, it would, I think, answer very well. We should then have to select the five appropriate sheets (each comprising four faces) and mount them on the umbrella stand; the five sheets would then represent one quarter of the globe.

The map-sheets may be kept in a very small compass, because the isosceles triangles may be folded down over the equilateral triangles, as shown in fig. 7.

In the other scheme it requires six or seven sheets to represent nearly one fifth of the globe: but it has the countervailing advantage of permitting a greater choice of the region which is to be in the middle; for, by having two umbrella stands, we can either place a pentagon or a hexagon in the middle.

In the instrument as made for the Geographical Society, the same general framework serves for both umbrellas, which may be shifted with great ease. It was this advantage in choice of the central region displayed which induced the Committee to prefer the original 32-faced polyhedron.

A similar construction may of course be applied to a dodecahedron inscribed in a sphere; and we thereby obtain a 72-faced surface, viz. 12 pentagons, and 60 obtuse-angled isosceles triangles. Here, as before, the map-sheets might be printed in sets of six, viz. a pentagon surrounded by five triangles; three map-sheets will then give one quarter of the globe. In this figure the pentagons are so large compared with the triangles that the approximation to the sphere is not very close.

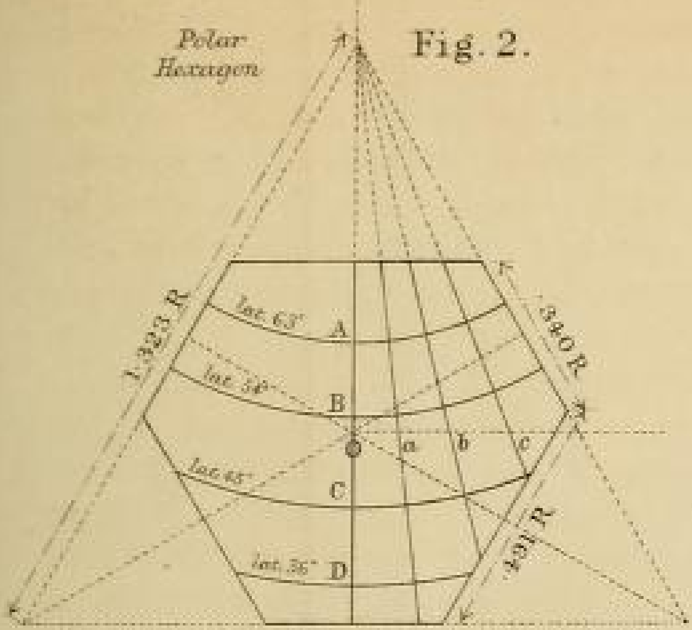
Models of the various plans explained were exhibited at Bradford; but, by an oversight, no abstract of this paper appeared in the British-Association Report.

LII. *On a new Vertical-Lantern Galvanometer.* *By* GEORGE F. BARKER, *M.D., Professor of Physics**.

DESIRING to show to a large audience some delicate experiments in magneto-electric induction, in a recent lecture upon the Gramme machine, a new form of demonstration galvanometer was devised for the purpose, which has answered the object so well that it seems desirable to make some permanent record of its construction.

Various plans have already been proposed for making visible to an audience the oscillations of a galvanometer-needle; but they all seem to have certain inherent objections which have prevented them from coming into general use. Perhaps the most common of these devices is that first used by Gauss in

* Communicated by the Author, having been read before the American Philosophical Society, May 7th, 1875.

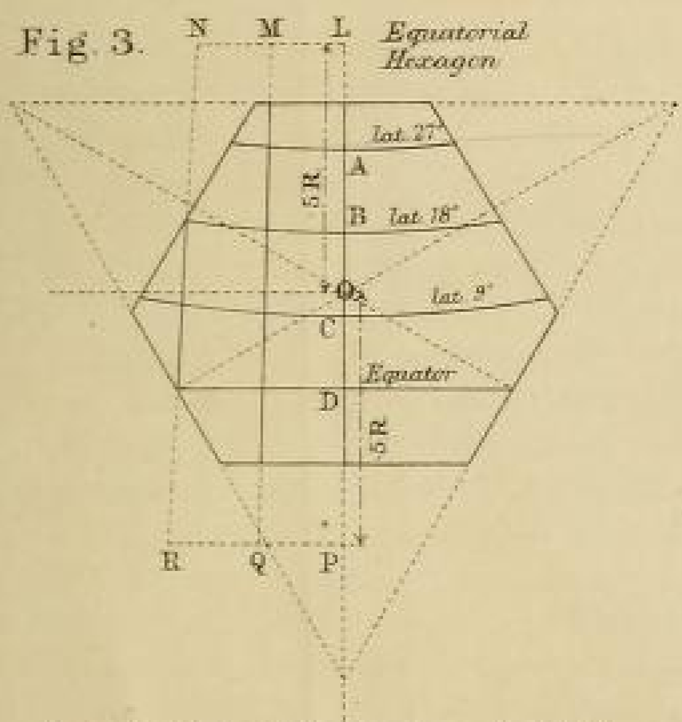


The radii of the circles of lat. given by R cot l.
The distances OA, OB, OC, OD by R cot (l + 37° 23′)
The distances Oa, Ob, Oc by R sin 37° 23′ tan γ
where γ = 9°, 18°, 27°
R is radius of inscribed sphere & l latitude.

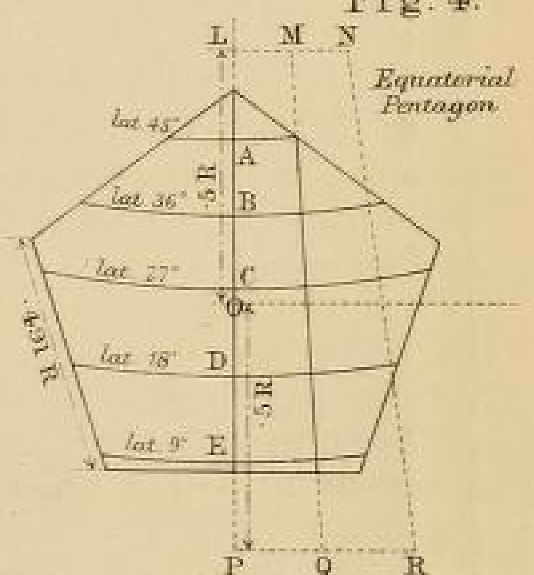
Fig. 1.

Diameter of inscribed sphere is 3 inches.

Fig. 3.

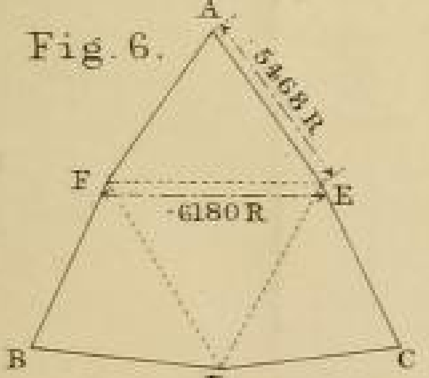


The radii of the circles of lat. given by R cot l.
The distances OA, OB, OC, OD by R cot (l + 79° 11′)
The intercepts LM, LN, PQ, PR given by R tan γ
(sin 79° 11′ ± ½ cos 79° 11′) where γ = 9° & 18°.



The radii of the circles of lat. given by R cot l.
The distances OA, OB, OC, OD, OE by R cot (l + 63° 26′)
The distances LM, LN, PQ, PR, by R (sin 63° 26′ ± ½ cos 63° 26′) tan γ where γ = 9° & 18°.

Fig. 6.



R is the radius of the circumscribing sphere.

Fig. 7.

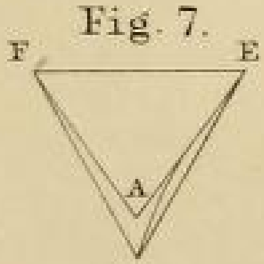
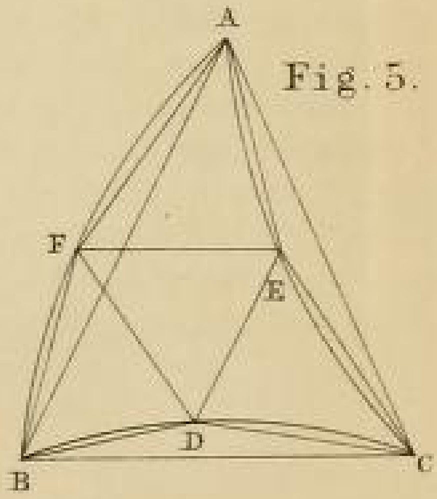


Fig. 5.



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