November 23, 1876.

Dr. J. DALTON HOOKER, C.B., President, in the Chair.

In pursuance of the Statutes, notice was given from the Chair of the ensuing Anniversary Meeting, and the list of Officers and Council proposed for election was read as follows :—

President.—Joseph Dalton Hooker, C.B., M.D., D.C.L., LL.D.

Treasurer.—William Spottiswoode, M.A., LL.D.

Secretaries.— { Professor George Gabriel Stokes, M.A., D.C.L., LL.D.
Professor Thomas Henry Huxley, LL.D., Ph.D.

Foreign Secretary.—Professor Alexander William Williamson, Ph.D.

Other Members of the Council.—Major-General John T. Boileau; Warren De La Rue, D.C.L.; Professor P. Martin Duncan, M.B., P.G.S.; Professor William H. Flower, F.R.C.S.; Professor Michael Foster, M.D.; Edward Frankland, D.C.L.; Francis Galton, M.A.; William Augustus Guy, M.B.; John Russell Hind, F.R.A.S.; The Rev. Robert Main, M.A.; William Pole, C.E., Mus. Doc.; The Rev. Bartholomew Price, M.A.; Rear-Admiral G. H. Richards, C.B.; Henry Clifton Sorby, Pres. Mic. Soc.; Professor Henry J. Stephen Smith, M.A.; Professor Balfour Stewart, M.A.

Mr. J. Croll and Prof. T. A. Thorpe were admitted into the Society.

The Presents received were laid on the table, and thanks ordered for them.

The following Papers were read :—

I. "On the Influence of Geological Changes on the Earth's Axis of Rotation" *. By George H. Darwin, M.A., Fellow of Trinity College, Cambridge. Communicated by Professor J. C. Adams. Received October 13, 1876.

(Abstract.)

The subject of the fixity or mobility of the earth's axis of rotation in that body, and the possibility of variations in the obliquity of the ecliptic, has of late been attracting much attention; but the author be-

* Since this paper was in manuscript Sir William Thomson has delivered his address to the Mathematical Section of the British Association at Glasgow. He therein touches on this subject, and gives some of the results attained here; but as he has not stated how he has attacked the problem, and as the subject has been recently attracting much attention, the author still ventures to offer his paper to the Royal Society.

lieves that it has not hitherto been treated at much length. The paper, of which the following is an abstract, is an attempt to investigate the results of the supposition that the earth is slowly changing its shape, with especial reference to the effects on the obliquity of the ecliptic and on the geographical position of the earth's axis of figure.

1. This part of the paper is devoted to the consideration of the precession and nutations of an ellipsoid of revolution which is slowly and uniformly changing its shape. The change is supposed to proceed from causes internal to the earth, and only to continue so long as the total changes in the principal moments of inertia C and A remain small compared to their difference, C—A.

The problem is treated by means of M. Liouville's extension of Euler's equations of motion of a rigid body about a point *. By an approximate method these equations may be treated as linear, and the solution divided into two parts.

Let θ be the obliquity of the ecliptic; Π cosec θ the precession of the equinoxes; $-n$ the angular velocity of rotation of the ellipsoid; $A+at$, $A+bt$, $C+ct$ the principal moments of inertia at the time t. Then it is shown that the secular effect on the obliquity of the ecliptic, as resulting from the motion of the principal axes in the body (which constitutes the first part of the solution), is given by the equation

$$\frac{d\theta}{dt}=-\frac{\Pi}{2n}\,\frac{a+b-2c}{A};$$

and as resulting from the change in the impressed forces, due to the change of shape of the body (which constitutes the second part), is given by

$$\frac{d\theta}{dt}=\frac{\Pi}{2n}\,\frac{a+b-2c}{C-A}.$$

The former part may be neglected compared with the latter. But from such geological changes as we are entitled to assume in the case of the earth, the total change in the obliquity of the ecliptic must be exceedingly small. Even gigantic polar ice-caps during the Glacial period could not have altered the position of the arctic circle by so much as 3 inches; and this is the most favourable redistribution of matter on the earth's surface for producing that effect. Thus the obliquity of the ecliptic has remained sensibly constant throughout geological history.

It is also shown that, during any gradual deformation of the ellipsoid, the instantaneous axis of rotation will always remain sensibly coincident with the principal axis of figure.

In the course of the work by which the previous results are attained there is shown to be a small inequality in the motion of the instantaneous axis, in consequence of which that axis describes a circle with uniform

* Liouv. Journ. 2e série, t. iii. 1858, p. 1; Routh's Rigid Dynam. p. 150.

velocity, and is coincident with the axis of figure every 306th day (in the earth). This circle touches the meridian along which the axis of figure is travelling in consequence of the deformation of the earth's shape. The diameter of the circle is shown in a particular case (not unfavourable to produce a large effect) to be less than $\frac{1}{376}''$. But although this inequality appears to be so small, it is of interest and is discussed at some length. It is shown that, if the earth be not quite rigid, this inequality might have the effect of modifying the path of the axis of figure in the body, in consequence of readjustments to a figure of equilibrium.

Various hypotheses as to the power of adjustment are considered, and the paths of the instantaneous and principal axes in the precession of a viscous spheroid undergoing deformation are found.

It is maintained that although the earth may be sensibly rigid to the tidally deforming forces exercised by the sun and moon, it would not be so to considerable departures from the figure of equilibrium, such as would arise from a wandering of the pole of figure from its initial position; and that readjustments to an approximate form of equilibrium probably take place, at considerable intervals of time, impulsively by means of earthquakes. Such periodical adjustments would not sensibly modify the geographical path of the principal axis as due to terrestrial deformation. But it is held that during the consolidation of the earth there must have been great instability in the geographical position of the poles. Throughout the rest of the inquiry, however, the hypothesis of the earth's sensible rigidity, together with the possibility of more or less rare impulsive readjustments to the figure of equilibrium, is adhered to. In consequence of these results dynamical considerations may be dismissed, and it only remains to consider the kinematical question of the change in the earth's principal axes due to any deformation of its shape.

2. Formulæ for this end are here found, and are adapted for numerical calculation. It is assumed, in the first place, that the deformation is such that there is no change in the strata of equal density; and accordingly all suppositions as to the nature of the internal changes accompanying geological upheaval and subsidence are set aside.

3. The forms of continent and depression are next investigated, which, for the transport of a given quantity of matter from one part of the earth's surface to another, would cause the maximum deflection of the principal axis of greatest moment—subject, however, to the condition that the layer excavated or piled up shall nowhere exceed a given small fraction of the earth's radius.

It is shown that the continents and depressions must be of uniform height and depth; there must be two of each, all similar to one another; that each has one of its own kind diametrically opposite to it; that they are in shape sphero-conics, formed by the intersection of a certain elliptic cone with the sphere; that the centres of the four sphero-conics are all on the same *complete* meridian and all in latitude 45°. A Table of numerical results depending on the values of certain elliptic functions is given.

4. In this part an endeavour is made to collect evidence as to the extent to which the earth may have undergone deformation from geological changes. The object is to discover what are the largest areas over which there has been a consentaneous rise or fall, and what is the greatest vertical amount of that rise or fall; also to determine how the erosion of the land and the sea affect the local excesses or deficiencies of matter on the earth's surface. The areas and amounts of elevation and subsidence which on a sealess and rainless globe are equivalent, as far as producing excesses or deficiencies of surface matter, to those which obtain on the earth are referred to as "effective;" and it is only the effective elevation or subsidence which we require to know in order to determine the shift of the earth's axis.

The evidence as to area is very meagre, because precise boundaries to regions of elevation and subsidence cannot be assigned; but, *faute de mieux*, the author's father, Mr. Charles Darwin, marked out for him on a map an area in the Pacific Ocean which (on account of the structure of the coral islands) he believes to have undergone subsidence within a recent geological period. From a consideration of this and of other points the author believes that from $\frac{1}{10}$ to $\frac{1}{20}$ of the whole earth's surface may, from time to time, have undergone elevation and subsidence. The greatest vertical effective amount of rise or fall cannot be determined from geological evidence, because of the effects of erosion and of the influx of the sea into parts below the mean level of the earth.

The only way of determining the point seems to be to find what is the difference of mass, standing on unit area of the earth's surface, in an ocean of, say, 15,000 feet deep, and in land of, say, 1100 feet high. From this difference of mass the effective elevation of an ocean-bed in its conversion into land can be at once determined. Taking the above numbers, it is found to be 10,436 feet; and in the examples given in the following part, the deflection of the polar axis, for an assumed effective elevation of 10,000 feet, is given in each case.

It is then pointed out that if the deformation of the earth were of very wide extent, the level surface of the sea would approximately follow the rocky surface, and that thus there might be sufficient change in the earth's shape to sensibly affect the position of the principal axis, without there being any geological signs of elevation or subsidence.

5. Numerical application is now made of the preceding work to the case of the earth, and, as before stated, all the results are given for 10,000 feet of effective elevation.

The first application is to continents and seas of maximum effect, and a Table of results is given. It may be here stated that if $\frac{1}{200}$ of the earth's surface is elevated, the deflection of the pole is $11\frac{1}{3}'$; if $\frac{1}{20}$,

$1° 46\frac{1}{2}'$; if $\frac{1}{10}$, $3° 17'$; and if $\frac{1}{2}$, $8° 4\frac{1}{2}'$ *. In each case an equal area is supposed to fall simultaneously.

Other examples are then given for continents and seas which do not satisfy the maximum condition; in some the boundaries are abrupt cliffs, in others shelving.

The conclusion is arrived at, that a single large geological change, such as those which obtain on the earth, is competent to produce an alteration in the position of the pole of from one to three degrees of latitude, on the hypothesis that there is no change in the law of internal density.

6. Various hypotheses as to the nature of the internal changes accompanying the deformation of the earth are discussed.

First, it is shown that if upheaval and subsidence are due to a shrinking of the earth as a whole, but to the shrinking being quicker than the mean in some regions and slower in others, the results are the same as those previously attained.

Second, the increase of surface matter due to the deposit of marine strata also gives the same results.

Third, the hypothesis that upheaval and subsidence are due to the intumescence or contraction immediately under the regions in question is considered. Under certain special assumptions, too long to recapitulate, it is shown that the previous results must be largely reduced. A Table of the values of the reducing factor for various thicknesses of the intumescent strata is given; from which it appears that if the stratum is tolerably thin and at all near the surface, the deflection of the pole is reduced to quite an insignificant amount. Even if the intumescence extends right down to the centre of the earth in a cone bounded by the elevated region, the results would be only about $\frac{2}{3}$ of the former ones. Hence it appears that the earlier results can only be stated as a superior limit to what is possible.

7. In conclusion it is pointed out that if the earth be quite rigid, no redistribution of matter in new continents could ever cause the deviation of the pole from its primitive position to exceed the limit of about 3°. But if the previously maintained view is correct, that the earth readjusts itself periodically to a new form of equilibrium, then there is a possibility of a cumulative effect; and the pole may have wandered some 10° or 15° from its primitive position, or have made a smaller excursion and returned to near its old place. No such cumulation is possible, however, with respect to the obliquity of the ecliptic.

It is suggested that possibly the glacial period may not have been really one of great cold, but that Europe and North America may have been then in a much higher latitude, and that on the pole retreating they were brought back again to the warmth. There seems to be, however, certain geological objections to this view.

* The area of Africa is about ·059, and of South America about ·033 of the earth's surface.