of the discharge, a change occurs in the heating of the electrodes. Such a phenomenon has been observed by Gassiot * at the discharge of a Grove battery of 400 pairs between metal or coke balls.

XXV. *On a suggested Explanation of the Obliquity of Planets to their Orbits.* By GEORGE H. DARWIN, *M.A., Fellow of Trinity College, Cambridge*†.

IN a former paper ‡ I have shown that if θ be the obliquity to its ecliptic of a planet which is slowly changing its shape, so that its principal moments of inertia at the time t are $\mathrm{A}+\mathrm{a}t$, $\mathrm{A}+\mathrm{b}t$, $\mathrm{C}+\mathrm{c}t$, then, so long as $\mathrm{a}t$, $\mathrm{b}t$, $\mathrm{c}t$ remain small compared with $\mathrm{C}-\mathrm{A}$,

$$\frac{d\theta}{dt}=\frac{\Pi}{2n}\frac{\mathrm{a}+\mathrm{b}-2\mathrm{c}}{\mathrm{C}-\mathrm{A}},$$

$\Pi\operatorname{cosec}\theta$ being the precession of the equinoxes, and $-n$ the rotation of the planet. This equation will hold true for long periods, if all the quantities on the right hand are treated as functions of the time; and if $\mathrm{a}=\mathrm{b}$ it may be written

$$\frac{d\theta}{dt}=-\frac{\Pi}{n}\frac{\frac{d}{dt}(\mathrm{C}-\mathrm{A})}{\mathrm{C}-\mathrm{A}}.$$

In the case of the earth,

$$\frac{6\pi^2}{n}\left\{\frac{1}{\mathrm{T}^2}+\frac{1}{\mathrm{T}'^2}\frac{1-\frac{3}{2}\sin^2 i}{1+\nu}\right\}\frac{\mathrm{C}-\mathrm{A}}{\mathrm{C}}=\frac{\Pi}{\sin\theta\cos\theta}=\frac{p}{n}\frac{\mathrm{C}-\mathrm{A}}{\mathrm{C}},\text{ suppose,}$$

where T, T' are the year and month, ν is the ratio of the earth's mass to the moon's, and i is the inclination of the lunar orbit to the ecliptic. In the corresponding function for any other planet there will be a term for each satellite, and $1-\frac{3}{2}\sin^2 i$ will be replaced by a certain function called λ by Laplace.

The equation may now be written

$$\frac{\mathrm{C}n}{p}\frac{d\theta}{dt}\log\tan\theta=-\frac{1}{n}\frac{d}{dt}(\mathrm{C}-\mathrm{A}).$$

The object of the present note is to apply this equation to the supposition that the planets were originally nebulous masses, and contracted symmetrically under the influence of the

* *Galvanismus* (2), Bd. ii. S. 1044.

† Communicated by the Author.

‡ "On the Influence of Geological Changes on the Earth's Axis of Rotation," Abstract, Proc. Roy. Soc. No. 175 (1876).

mutual gravitation of their parts. This application involves a large assumption, viz. that the precession of a nebulous mass is nearly the same as though it were rigid. In defence thereof I can only quote Sir W. Thomson, who says, "Now, although the full problem of precession and nutation, and what is now necessarily included in it—tides, in a continuous revolving liquid spheroid, whether homogeneous or heterogeneous, has not yet been coherently worked out, I think I see far enough towards a complete solution to say that precession and nutations will be practically the same in it as in a solid globe, and that the tides will be practically the same as those of the equilibrium theory" *.

I therefore once for all make this assumption.

The coefficient p depends solely on the orbit of the planet and of its satellites, and during the contraction of the mass will have been constant, or very nearly so. To determine the other quantities involved, we have the three following principles:—

(1) The conservation of angular momentum.
(2) The constancy of mass of the planet.
(3) That the form of the planet is one of equilibrium.

(1) is expressed by the equation $Cn=H$, a constant; and, if ρ, a be the mean radius and density of the planet at any time, (2) by $\frac{4}{3}\pi\rho a^3=M$, the mass. Then, if the law of internal density during contraction be that of Laplace, viz. $\frac{Q\sin qr}{r}$, if k be the ratio of the surface-density to the mean density, e the ellipticity of the surface, and m the ratio of the centrifugal force at the distance a to mean pure gravity, the third principle gives †

$$\frac{5m}{2e}=\frac{(qa)^2}{3k(qa-1)}-3k.$$

Also

$$C=\frac{2}{3}\left\{1+\frac{6}{(qa)^2}(k-1)\right\}Ma^2,$$

$$C-A=\frac{2}{3}\left(e-\frac{m}{2}\right)Ma^2,$$

$$m=\frac{3n^2}{4\pi\mu\rho}.$$

* Address to Section A. of the British Association at Glasgow, 'Nature,' September 14, 1876, p. 429.

† Compare Thomson and Tait's 'Natural Philosophy,' § 824 (14), § 827 (20).

Hence (1), (2), and (3) lead to the following equations:—

$$\frac{2}{3}\left\{1+\frac{6}{(qa)^2}(k-1)\right\}Ma^2n=H, \quad . \quad . \quad . \quad (4)$$

$$\rho a^3=\frac{3M}{4\pi}, \quad . \quad . \quad . \quad (5)$$

$$\frac{n^2}{4\pi\mu\rho}\left\{\frac{5}{\frac{(qa)^2}{3k(qa-1)}-3k}-1\right\}Ma^2=C-A. \quad . \quad . \quad (6)$$

If during contraction qa remains constant, and if the coefficient of Ma^2n in (4) be called γ, and that of $\frac{Ma^2n^2}{4\pi\mu\rho}$ in (6) be called β, then it will be found that

$$\frac{1}{n}\frac{d}{dt}(C-A)=-\frac{H\beta}{\gamma}\frac{1}{12\pi\mu\rho^2}\frac{d\rho}{dt}.$$

Hence, remembering that $Cn=H$,

$$\frac{d}{d\rho}\log\tan\theta=\frac{p\beta}{12\pi\mu\gamma\rho^2}.$$

Integrate, and let D, I be the present values of ρ and θ; then

$$\log\frac{\tan\theta}{\tan I}=\frac{p\beta}{12\pi\mu D\gamma}\left(1-\frac{D}{\rho}\right).$$

If we assume that qa has always the same value as it now has in the case of the earth *,

$$\gamma=\cdot3344,\ \beta=\cdot9507,\ \text{and}\ \frac{\beta}{\gamma}=2\cdot8433.$$

If during contraction the planet were always homogeneous, the factor $\frac{\beta}{\gamma}$ would be replaced by $\frac{15}{4}$, or 3·75.

Let K stand for 2·8433, or 3·75, as the case may be; let $Q=\frac{1}{T^2}+\Sigma\frac{\lambda}{T'^2(1+\nu)}$; let P be the periodic time of a pen-

* In determining the precessional constants of Jupiter and Saturn, Laplace assumed that their law of internal density was the same as that of the earth. The assumption is, I believe, unjustifiable; but it will give sufficiently good results for the present purpose. The limiting value of $\frac{\beta}{\gamma}$, when the surface-density is infinitely small, and if the Laplacian law still holds good, is 1·99. See 'Monthly Notices of the Royal Astronomical Society,' December 1876.

dulum of length equal to the present mean radius of the planet, swinging under mean pure gravity. Then

$$\frac{p}{2\pi\mu\mathrm{D}}=\frac{\mathrm{P}^2\mathrm{Q}}{6},$$

and the equation becomes

$$\log\frac{\tan\theta}{\tan\mathrm{I}}=\frac{\mathrm{KP}^2\mathrm{Q}}{6}\left(1-\frac{\mathrm{D}}{\rho}\right).$$

This equation shows that as ρ diminishes θ diminishes, and when ρ is infinitely small θ is zero. That is to say, if a nebulous mass is rotating about an axis nearly perpendicular to the plane of its orbit, its equator tends to become oblique to its orbit as it contracts.

In the case of the earth, $\mathrm{P}^2\mathrm{Q}=\frac{8\cdot5577}{10^8}$; and taking the present obliquity of the ecliptic as 23° 28′, the equation may be written

$$\mathrm{Log}_{10}\tan\theta=9\cdot63761-\frac{1\cdot7612}{10^8}\cdot\frac{\mathrm{D}}{\rho}$$

On the hypothesis of homogeneity, 1·7612 must be replaced by 2·3229.

The extreme smallness of the coefficient of $\frac{\mathrm{D}}{\rho}$ shows that the earth must have had nearly the same obliquity even when its matter was rare enough to extend to the moon. But if it can be supposed that the moon parted from the earth without any abrupt change in the obliquity of the planet to the ecliptic, then from that epoch backwards the function Q would have had only one term, viz. $\frac{1}{\mathrm{T}^2}$, and $\mathrm{P}^2\mathrm{Q}$ would be $\frac{2\cdot5750}{10^8}$. The coefficient of $\frac{\mathrm{D}}{\rho}$ in the above equation would be reduced to $\frac{5\cdot30}{10^9}$, or $\frac{7\cdot00}{10^9}$, according to whichever value of K is taken. This being granted, it follows that when the diameter of the earth was 1000 times as large as at present, the obliquity to the ecliptic was only a few minutes.

This somewhat wild speculation can hardly be said to receive much support from the cases of the other planets; but it is not thereby decisively condemned. In all the planets up to and inclusive of Jupiter, the expression Q will have to be reduced to its first term $\frac{1}{\mathrm{T}^2}$, because the satellites are rather near their primaries. Hence one would expect that the obliquities of the planets to their orbits would diminish as we go away from the sun. It is believed (but the observations seem doubtful) that Mercury and Venus are very oblique to their orbits ; and Mars has an obliquity nearly the same as that of the earth. The region of the asteroids is a blank ; and then we come to Jupiter, with a very small obliquity.

The next in order is Saturn : and his case is unfavourable ; for he is slightly more oblique to his orbit than is the earth. Nevertheless it must be observed that he has a large number of satellites, and some are very remote from him, and his mean density is very small ; hence, if the satellites can have affected the obliquity in any case, one would expect them to have done so in that of Saturn.

No light whatever is thrown on the case of Uranus, whose axis is said to lie nearly in the plane of his orbit.

XXVI. *On the Magnetization of Steel by Currents.* *By* E. Bouty, *Docteur ès Sciences.*

[Concluded from p. 135.]

III. *Temporary and Permanent Magnetization of thin Needles only slightly hardened.*

THESE experiments, like the foregoing, were made with the steel wire used for spindles in clock-making ; but instead of steeping the needles at a red-heat, they were used in the state in which they were delivered by the maker—that is to say, not tempered. Here rupture-experiments are out of the question, and it is convenient to take as a starting-point, not the study of the permanent magnetization, but that of the temporary.

The needles to be investigated are placed at a distance from the galvanometer-needle equal to 50 or 60 centims. ; and the operation is conducted according to the method employed in the preceding section*. At this distance, and with the dimensions of the needles used, Gauss's formula applies without needing the introduction of the corrective terms, and one can compare the moments of needles of different lengths without knowing any thing *à priori* of the position of the poles. Let us consider n needles of different lengths $l, l', \ldots$, and submit

* The quantity of magnetism being much greater in feebly hardened needles than in the same needles strongly hardened, the employment of long distances does not diminish the absolute values of the deflections sufficiently to render the measurements uncertain in the present case, as it did in that of the preceding section.