

The following papers were announced :—

*J. Tebbutt.* Observations of Brorsen's Comet, February and March 1879.

*Dr. T. Oppolzer.* Determination of the Longitudes of Berlin, Munich, Leipzig, Vienna, Paris, and Pulkowa.

*C. Carpmael.* On the values of the constants in the equation  $rA_r x^{(r)} + rA_{r-1} x^{(2-1)} + \dots + rA_1 x^{(1)} + \dots + rA_0 - y_x = 0$ , obtained by the method of least squares, from the  $n+1$  values of  $y_x$  when  $x=0, 1, 2 \dots n$ ;  $n$  being greater than  $r$ .

*D. Gill.* On the value of the Solar Parallax derived from the observations of Mars, made at Ascension during the opposition of 1877.

*G. M. Seabroke.* Spectroscopic observations of the motion of Stars in the line of sight, made at the Temple Observatory, Rugby.

*E. J. Stone.* Note on the Cape Catalogue of 12450 stars. (Extract from a letter to the Astronomer Royal.)

*R. H. M. Bosanquet and Prof. A. H. Sayce.* Preliminary note on the Babylonian Astronomy. The Babylonian Calendar.

*A. Marth.* Note on the transit of the Earth and Moon across the Sun's disk, as seen from Mars on Nov. 12, 1879, and on some kindred phenomena.

*Rev. J. Pearson.* On a new method of Mounting an Equatoreal.

The Meeting adjourned at ten o'clock.

### *A Tidal Theory of the Evolution of Satellites.*

If a planet were formed of fluid it would assume a spheroidal shape under the influence of its rotation; but if a satellite revolves in a circular orbit round the planet in the plane of the equator, tides will be raised in the planet such that the spheroid will become distorted into an ellipsoid with three unequal axes, and the longer axis of the equator will always be directed towards the satellite. Thus the shape of the planet revolves along with the satellite, whilst each particle of fluid rotates with the planet, and has therefore to rise and fall twice in every revolution of the planet relatively to the satellite.

Now suppose that at any instant the fluid becomes endowed with friction; then those particles of fluid which happen at that instant to be in the equatoreal protuberances cannot fall so quickly as they did before the friction began to act. Thus the particles linger in the protuberances longer they than should do, and the particles which ought to be rising do not rise so quickly as they should. The obvious consequence of this is that the equatoreal protuberances and depressions are carried on along with the planet's rotation, and the longer axis of the equator now points in advance of the satellite in its orbit. This condition is perpetuated as long as

the friction lasts. Tides of this kind are said to lag; and the angle of lagging may be defined as the angle between the longer axis of the equator and the direction of the satellite.

Another result of the supposed internal friction or viscosity of the fluid is, that the oscillations of the particles are impeded, and that they have not time to move so far as they would have done without friction. Hence if the fluid be viscous, the tides not only lag but are reduced in height.

A calculation of the effects of viscosity shows that in a mass like the earth we must suppose an enormous degree of internal friction, before the tides are much reduced or lag considerably. Indeed to have much effect in altering the tides, the matter constituting the planet must be of so stiff a consistency, that it would be called a solid for ordinary purposes. In the theory of which I am here giving a short account, a viscosity, which must be regarded as not at all great, is such that, if we conceive a slab of the material an inch thick to have one of its faces held fixed, whilst to the other face is applied tangentially a force of 13 tons to the square inch, then after the lapse of 24 hours the upper face will be shifted relatively to the lower through  $\frac{1}{10}$  of an inch. As far as our knowledge of the interior of the earth goes, it might quite well be formed of matter of a stiffness comparable with this. There is, however, reason for believing that the earth is now very much more rigid than this; for if its mass yielded tidally to any great extent, then the ocean tides would be largely reduced; and although the theory of ocean tides is as yet very incomplete, yet we know that the height of the tides cannot be very much less than it would be on a rigid nucleus\*.

Although the bodily tides are probably now inconsiderable, yet there is much reason to believe that the earth was once a hot molten mass; and therefore it is probable that, in the course of its cooling, the materials of which it is constituted must have passed through all degrees of viscosity, from approximate fluidity to its present high rigidity. This, therefore, seems to justify the theory of which we are speaking.

We will now return to the general theory of a viscous planet and its satellite; and, in order to obtain a convenient phraseology, the planet and satellite will henceforth be spoken of as the earth and moon.

The next point to be considered is one in which the lagging of the tides shows its most important influence. If the fluid forming the earth were frictionless, the equatorial protuberances would be in a straight line with the moon; but in consequence of the friction they are not in that line, and therefore the moon exercises forces on the protuberances which tend to bring them back. Then since the protuberant axis of the equator always points in advance of the

\* See Sir W. Thomson's investigation of elastic tides (Thomson and Tait's *Nat. Phil.*), and my paper on viscous tides, *Phil. Trans.* 1879, part i.

moon, therefore the moon is always putting a brake on to the earth's diurnal rotation, and conversely the earth is always accelerating the moon. The first result of this is that the diurnal rotation is retarded. The second effect is that the moon's linear velocity is accelerated; this acceleration clearly will prevent the moon from moving in a path of so sharp a curvature as before, and hence the size of the circle described by the moon round the earth is continually increasing. But then the moon cannot get round the larger circle so quickly as round the smaller one; and hence her angular velocity is retarded, although her linear velocity is augmented.

The two results are therefore a lengthening of the day and of the month; in the present state of the earth and moon the rate of alteration of the day would be a good deal greater than that of the month.

Now a terrestrial observer, who did not know that the earth is not a perfect time-keeper, would suppose that the moon was being accelerated, because the earth's rotation is slackening; on the other hand, a part of this apparent acceleration would be masked by the fact that the moon is really going slower. On the whole, taking the earth as our clock, there is an apparent acceleration of the moon's motion.

This was pointed out 20 years ago by Delaunay and Adams; and they have referred to this cause an otherwise inexplicable, although very small, secular acceleration of the moon's mean motion. They indeed attributed the friction to the oceanic tides; but there seems to be no reason why it should not be, at least partly, due to bodily tides in the earth, such as we are here considering.

Then adopting the theory of bodily tides as the sole explanation of this observed secular acceleration, we may obtain a numerical relation between the height and lag of the tides.

This relation shows that either the tides lag by a very small angle, or that their height is very small. The small lag would correspond to approximate fluidity or nearly perfect elasticity of the earth's mass; and the small height may correspond to either very great viscosity, or to elasticity approaching perfect rigidity. Then as the earth is not nearly fluid, and as it probably does not yield largely as a nearly perfectly elastic body, it is probable that, whether nearly perfectly elastic or very viscous, it possesses a very large effective rigidity. This conclusion is therefore confirmatory of that deduced from the ocean tides.

There is nothing here, however, to negative the hypothesis that in past ages the rigidity may have been not nearly so great, and the theory of viscosity still seems quite legitimate.

We will now assume that the earth is viscous, and will trace the configuration of the two bodies backwards in time. In the present state the rate of diminution of the earth's angular velocity of rotation is much greater than that of the moon's orbital angular velocity; if the former were exactly  $27\frac{1}{3}$  times as great as the latter then the day and month would (in the retrospect), in a given time be

reduced in the same proportion. Now this, to a very rough degree of approximation, is the present proportion, and accordingly in the retrospect we find that when the day has fallen to  $15\frac{1}{2}$  hours the month is 19 days long, so that each of the two has been reduced to about  $\frac{2}{3}$  of its primitive value.

But the further back we go the less true is this proportionality, the month being much more rapidly reduced than the day, so that when the day has fallen to 6 hours 50 minutes the month is only 1 day 14 hours. With a certain assumed viscosity of the earth, namely that aboved defined, I found that this enormous change might have taken place in 56 million years.

If the changes be traced still further back, we ultimately come to a state where the month and day are identical and are both  $5\frac{1}{2}$  hours long. In this condition the moon was always opposite the same face of the earth, and there were only 5000 miles intervening between the surfaces of the two bodies. The conditions of the problem are such that it is not possible to give this period and this distance with any great accuracy, and it seems fairly probable that a more accurate solution would show the two bodies still more nearly in contact.

Now the conditions of stability of rotating fluid masses are very obscure; but it seems that if in consequence of rapidity of rotation a fluid planet could no longer subsist as a single body, and were gradually to separate into two, then immediately after separation the condition of the two masses would be closely similar to that to which we have traced back the earth and moon.

Hence there seems a considerable probability that the moon and earth once formed parts of a common mass, and that they attained their present configuration through the action of the frictional tides raised in the earth by the moon and in the moon by the earth.

In this retrospect we have found the day and month converging to identity; but if we look onward in time we find them again converging to a common period of about 50 of our present days\*. One rather curious consequence of this is that the number of terrestrial revolutions in a lunar period can never exceed a certain amount; and it appears that this maximum is about 29, and that we have already passed through that phase. Thus in one sense tidal friction has done more than half its work.

We have hitherto supposed the earth's axis to be perpendicular to the lunar orbit; but in the actual investigation the obliquity of the ecliptic was tracked *pari passu* with the lengths of the day and month.

If the axis of the earth be oblique to the lunar orbit, the tides are complicated by the variation in the disturbing forces with the moon's declination. The tides may then be analysed into seven different waves, which are propagated with different velocities; or

\* See Thomson and Tait's *Nat. Phil.* § 276.

these, three have a period of nearly a half-day, three of nearly a day, and one has a period equal to a semi-period of the moon or fortnight. One of the semi-diurnal, one of the diurnal, and the fortnightly tide are very small; and of the rest the "lunar semi-diurnal tide" is by far the most important.

In an article of this nature, it would be out of place to attempt to show how the attraction of the moon on the several tides affects the earth's axis of rotation\*. It must here suffice to state that the obliquity of the ecliptic will increase if the viscosity of the earth's mass is less than a certain amount, but will decrease if it be greater. Hence it is most probable that at the present time the obliquity is diminishing, but with such slowness that thousands of years must elapse before the decrease can become sensible.

On the other hand, it is probable that in the earlier stages of the earth's history, when the earth was more nearly molten, the obliquity was increasing.

In order to determine how far this theory can go towards explaining the obliquity of the ecliptic, I took the most favourable case, viz. where the viscosity is small, and traced backwards in time the obliquity, and the lengths of the day and month from their present values. Of the day and month we have already spoken; but the obliquity was found also to diminish in the retrospect, so that when the day was 6 hours and the month 12 hours, the obliquity had decreased from the present  $23\frac{1}{2}^{\circ}$  to about  $14^{\circ}$ .

It is certain, however, that this is to some extent an overstatement, because the simultaneous changes of the inclination of the lunar orbit to the ecliptic were neglected, and I have reason to believe that in the earlier times that inclination might perceptibly modify the results. The changes of obliquity were not traced backwards beyond this point, because no account was taken of these parallel changes of inclination of the lunar orbit.

The fact that the obliquity ceases to vary when the month is twice the day, arises from a general theorem, that if the planet have a small viscosity, and if the period of the satellite is twice that of the planet's rotation, the rate of change of obliquity vanishes.

On the whole I conclude that some  $8^{\circ}$  or  $9^{\circ}$  of obliquity may probably be referred to the action of the tides.

In a second paper† I have considered a secular distortion of the earth's mass which must be a result of tidal friction.

The action of the moon on the lagging tides has been already likened to a brake. Now it appears that by far the larger part of the brake power is applied in the equatorial regions; and as the earth is by hypothesis viscous, it follows that the rotation of the equatorial regions must be retarded rather more quickly than that of the polar regions. Hence there must arise a screwing motion of

\* Some details are given in the abstract of a paper on "The Precession of a viscous Spheroid," in the Proc. R. S. No. 191 (1878).

†Of which an abstract will be found in the Proc. R. S. No. 191 (1878).

the earth's mass, the polar regions shifting from west to east relatively to the equator.

Numerical calculation shows that the rate of distortion must have been so slow in latter times, that no perceptible effect can have thus arisen within geological history. It is, however, just possible that the N. and S. trend of the great continents may be connected with this cause, for the continents may be the great wrinkles left by former twisting.

In a second part of the paper the amount and distribution of the heat generated within the earth by friction is investigated. It appears that in the course of the prolongation of the day from  $5\frac{1}{2}$  hours to 24 hours, enough heat must have been generated within the earth to heat the whole mass  $3000^{\circ}$  Fahr., supposing it to have the specific heat of iron and the heat to be all applied at once. It would seem at first sight that we have here an explanation of the observed increase of the temperature of the earth as we go downwards; but a more careful examination of the question shows that only a very small fraction of that increase can possibly be referred to this cause.

The dynamical investigation, of which a slight sketch has now been given, yields results which obviously point to a great modification of the nebular hypothesis of Laplace and Kant. It would as yet be premature to examine the solar system to see how far this tidal theory of evolution can explain the configuration of planets and their satellites; for it remains to investigate the secular changes of the inclinations and eccentricities of the orbits of satellites due to tidal reaction. I hope in a few months' time to have completed at least a partial investigation of these points; and the results already attained appear of considerable interest.

The proof or disproof of the theory will turn on numerical results, and to obtain those results in the cases of other planets will entail much work. At present I will only remark that the anomalous satellite of Mars, which revolves faster than its primary rotates, appears to present a case strongly confirmatory of these views; for it seems probable that its extreme minuteness has preserved it as a standing memorial of the primitive time of rotation of Mars round his axis. On the other hand, the system of Uranus is very perplexing; and so little is really known about that planet and his satellites, that it will be difficult to put the theory to the test.

G. H. DARWIN.

### *Occultation of Antares.*

THE rarity of occultations of bright stars at any given place is well illustrated by the case of  $\alpha$  Scorpii, this star having been covered by the Moon, as seen from some point on the Earth's surface, at every lunation for the past four or five years; whereas only once, viz. on the evening of July 28 next, between  $9^{\text{h}} 37^{\text{m}}$  and  $10^{\text{h}} 7^{\text{m}}$ , is the