

12° C., Eötvös* 73·06). This agreement seems to show that the tension of a water-surface already only 0·06 second after the formation of the surface (and according to what is discussed in the present paper probably much earlier) has assumed the constant value which the tension, if contaminations are kept away, will retain during a very long time.

The Tidal Observations of the British Antarctic Expedition, 1907.

By SIR GEORGE DARWIN, K.C.B., F.R.S.

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The present investigation was undertaken at the request of Sir Ernest Shackleton; the expense of the reduction was defrayed by him, and this paper is now communicated to the Royal Society by his permission. It will ultimately be republished as a contribution to the volume of the physical results of the expedition.

The first section, describing the method of observing, is by Mr. James Murray. The second section explains the reduction of the observations and gives a comparison between the new results and those obtained by the "Discovery" in 1902–3. The third section is devoted to the discussion of certain remarkable oscillations of mean sea-level and to speculations as to their cause and meaning.

I.—ON THE METHOD OF OBSERVING THE TIDES.

Early in June, 1908, preparations were begun for the erection of a tide-gauge, the most important feature of which was to be a recording apparatus made from a modified barograph. Owing to various delays and mishaps it was not before the middle of July that the gauge was completed in its final form, and the continuous record begun, which was carried on for more than three months, subject only to the loss of half an hour weekly, while the paper was being changed.

Dr. Mackay undertook the erection of the instrument, the apparatus was devised by the joint suggestions of Messrs. David, Mackay, Mawson, and Murray, while Mr. Day did the more delicate part of the work, namely, the alteration of the barograph.

* 'Math. es Természettud.,' 1885, vol. 3, p. 54 (Budapest).

The diagram (fig. 1) shows the chief parts of the apparatus and their relations to one another. The ice is shown in section, with the tripod and recording apparatus erected on it. A weight A, consisting of a box filled with stones, rests on the sea bottom. A piece of iron tubing B is let through the ice vertically and fastened. It is filled with paraffin oil, the object of which is to prevent the wire being frozen in, an idea used with success by the officers of the "Discovery." A wire C is taken from the weight on the sea bottom, passed through the oil-filled iron tube B, over the pulley D, and fixed to the end of the bamboo lever E, where it is kept

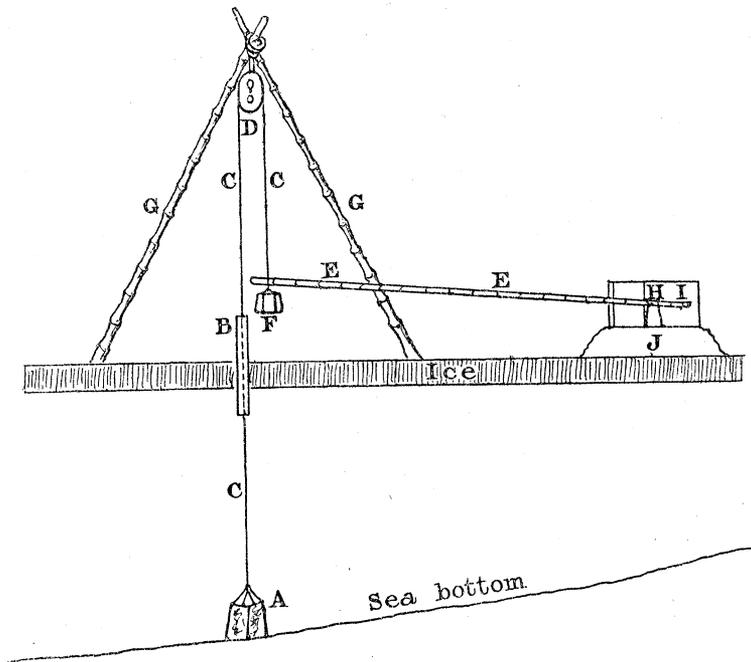


FIG. 1.

taut by the smaller weight F. The pulley D is suspended from a tripod of bamboo poles, of which two legs G are shown. The long lever E works on a spindle at H, and its short end I is connected by a cross-piece with the pen of the barograph. The details of this part are too small to be shown in this diagram, and will be illustrated in another figure. From this diagram there are omitted several parts, such as the guides which prevent the long lever from swinging during a blizzard, which are not essential to the understanding of the instrument. The barograph was of necessity covered by a box to keep out the snow. The lever entered through a slit in the end of the box, and an arrangement of canvas kept the snow out. The box containing

the barograph was raised on a little mound of snow J, in order to give the lever equal play above and below, or in other words, to allow of the mean sea-level being recorded about half-way up the drum. Of course the mean level had to be ascertained by a little observation.

The second diagram (fig. 2) is a plan on a larger scale of the recording part of the apparatus. The circle A is the drum of the barograph; B is the pen making the tracing on the drum; C is the axle on which the lever bearing the pen works. This lever is continued beyond the axle to a distance rather greater than that of the part bearing the pen. This end of the lever D is made much heavier than the other. The long bamboo lever E, of which only a small part is shown, is borne on an axle F, which is in line

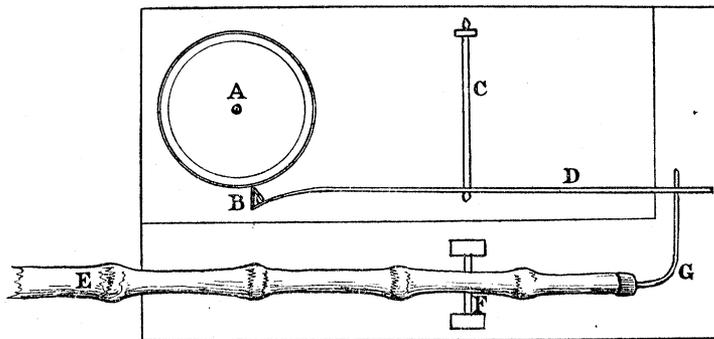


FIG. 2.

with the axle C in the barograph, though of course quite unconnected with it, being outside the glass box of the barograph. Attached to the end of the bamboo is a stout wire G, bent round so that it passes under the end D of the pen lever, which rests upon it by its own weight, and rises and falls with it, but, being quite free from it, is not affected by any vibration of the bamboo under the influence of the wind. The barograph pen has, of course, been uncoupled from the aneroid capsules, which are not indicated in the plan.

Dr. Mackay, with much assistance from Prof. David, had the tide-gauge set up, all but the recording part, by June 22. In order to utilise the facilities we now had for noting the changes of level, while waiting for the recording instrument to be finished, Dr. Mackay devised a very simple arrangement for ascertaining the amount of the tide. It was simply an inclined plane on which a paper marked with lines an inch apart was pinned. On this there slid a heavy block of wood which was attached to the end of the wire coming over the pulley. A lead pencil was inserted through a hole in the block of wood, which was kept in position by two guides.

This arrangement is shown in fig. 3, which is drawn in perspective. The pulley A is suspended from the tripod B, B, B. The wire C is attached to the wood block D, which slides on the inclined board E between the guides F, F. The pencil G is fixed so as to project a little below the block of wood; H is the line traced by the pencil.

This simple device was not intended to give anything but a straight line but it was hoped by frequent inspections to ascertain the turn of the tide,

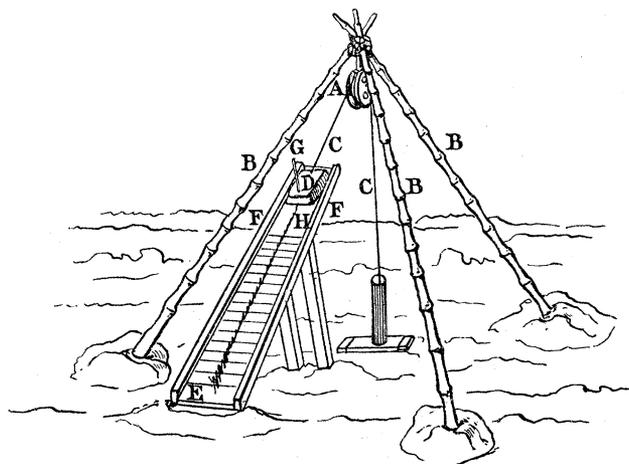


FIG. 3.

and Dr. Mackay kept vigil one night, and visited the gauge at intervals of about an hour.

Owing to a general slackness of the parts of the instrument the line traced was not a straight line, but a zigzag one, which proved of much greater interest. The pencil in descending did not follow the same course as it made going up, but swerved a little, and thus we got the first indications of what we believe to be seiches, at any rate of regular oscillations of much shorter intervals than those of the tides. The tracing obtained thus accidentally was too vague to enable us to count with certainty the number of periods per hour, but at any rate it demonstrated oscillations of a period of a few minutes and an amplitude of a few inches.

A number of records were taken in this rough manner till July 3, when the wire was found to be broken. A new situation was then selected for the tide-gauge, nearer the house, about 100 yards from shore, where the depth was 13 fathoms. The ice being now of considerable thickness, it was with no little labour that Dr. Mackay, with the help of Mr. Marston, got a new hole made to put the weight down.

By July 8 the apparatus was completed at the new place, this time with the recording attachment. A preliminary record was got from July 8 to 11, and the instrument was then stopped for readjustment. On July 14 it was finally started, and ran without mishap till nearly the end of October.

Prof. David usually changed the papers weekly. It was impossible to do this in the field, as it required bare hands. The barograph was therefore disconnected and carried to the house, where a new paper was put on and fresh ink put in the pen. The whole changing did not take more than half an hour.

The scale on which the curve was traced was about one-nineteenth of actuality—the long end of the lever being 11 feet and the short end 7 inches. The value of the factor of reduction of amplitude has therefore been taken to be $7/132$ or $1/18.857$.

One of the first records for a complete week was analysed. The curve appeared a simple one with one maximum daily, but a slight flattening of the minima indicated that other elements were present. The analysis showed that there was a smaller tide, having two maxima daily. The whole range of the tide at its highest was about 3 feet. The greatness of it surprised us, as the tide cracks usually showed a difference of level of not more than from 1 foot to $1\frac{1}{2}$ feet. This may have been because the free edge of the crack had not room to sink to the full extent of the tide, but came on bottom, and the ice then sagged away to the level part.

Towards the end of October the tripod was blown down during a blizzard, and the wire was snapped. The ice was by this time so thick that it was found impracticable to cut another hole to put a weight down, and so the observations were discontinued.

The curve traced on the drum gave very frequent indications of seiches in the form of festooning, but the scale was so small and the clock-motion so slow that these indications were blurred and useless for study. It was intended, and would have been easy, to substitute a clock of about 10 times the speed belonging to a Callendar recorder which we had with us, but it was late in the season before we could try it, and the breaking of the wire put a stop to the attempt.

II.—THE REDUCTION OF THE TIDAL OBSERVATIONS.

The motion of the ice carrying the tide-gauge relatively to the sea-bed was transmitted to the drum by means of a lever, as explained above. Accordingly, it is very nearly exactly the chord of the arc turned through by the lever which ought to have been measured. Yet it is the arc itself which is recorded on the curvilinear scale on the drum. The angle turned through

by the lever is, however, sufficiently small to permit us safely to neglect the correction in strictness required for the conversion of arcs to chords, and the arcs have been accepted as giving the changes of water level with sufficient accuracy.

The tidal record extended from July 14 to October 25, 1908, but the sheet which bore the record from October 12 to 18 is missing, and the record actually treated ends with October 11.

It was possible by means of a few interpolations to obtain an unbroken record from 0 h. astronomical time of July 14 to 23 h. of October 11. About an hour was generally lost once a week, while the paper was being changed, but it was always easy to complete the curve over this short interval by a pencil line, and this was regarded as equivalent to the actual curve.

On September 13 the pen failed to mark, but the curves on the 12th and 14th were unusually regular in character, so that a good interpolation for the 13th was easily obtained. The following is a list of the interpolated readings, and the hours are given inclusively in astronomical time:—

July 14, 0 h. to 3 h. (extrapolated); July 19, 19 h. to 23 h.; September 12, 17 h. to 23 h.; September 13, 0 h. to 23 h.; October 11, 22 h. and 23 h.

The errors of the clock do not seem to have been great enough to demand attention, and in fact, they are not always noted on the diagrams.

The clock was kept to apparent time, and was reset as the equation of time changed sensibly. But the scheme of reduction assumes that mean time has been used. This error may be taken into account with sufficient accuracy by certain changes in the true longitude of the place of observation, which was $166^{\circ} 12' E$.

The observations were broken into three groups of a month each, for which the epochs were: (1) 0 h., July 14; (2) 0 h., August 13; (3) 0 h., September 12, 1908.

To allow for the equation of time the longitude for the first month was taken as 6 m. of time or $1^{\circ} 30'$ further east than in reality; in the second month the longitude was regarded as correct, and in the third it was shifted 10 m. or $2^{\circ} 30'$ to the west. The correction for the last month is less satisfactory than for the other two, because at that time of year the equation of time is changing rapidly, and differs considerably at the beginning and end of the month.

The unit adopted in tabulating the height was $1/10$ of an inch of the scale on the drum. Since 1 inch on the drum corresponds to 18.857 inches of water, the heights as derived from the harmonic analysis of the drum readings were converted to inches on multiplication by 1.8857.

For the tides M and O the observations were also treated as appertaining to a single period of three months, without regard to the equation of time; and a similar treatment was also extended to the tides S, K₂, K₁, and P₂ as will be explained more fully hereafter.

The reductions were made, under my supervision, by Mr. F. Finch with my apparatus,* and in the first instance the three months were discussed independently. The semidiurnal tides were derived from months of 30 days, and the diurnal tides from months of 27 days. In this treatment it is necessary to assume that the phase of the tide K₂ is the same as that of S₂, and that the amplitude of K₂ is 3/11ths of that of S₂. Similarly, we must assume identity of phases for K₁ and P, and that the amplitude of P is 1/3rd of that of K₁.

The following are the results:—

	1. July 14— Aug. 11.	2. Aug. 12— Sept. 11.	3. Sept. 12— Oct. 11.
M ₂	H = 2·55 in. κ = 357°	2·64 in. 11°	2·11 in. 5°
S ₂	H = 1·08 in. κ = 293°	1·21 in. 282°	1·20 in. 267°
K ₂	H = 0·29 in. κ = Same	0·33 in. as for K ₂ .	0·33 in.
K ₁	H = 7·66 in. κ = 6°	8·57 in. 9°	10·06 in. 10°
P	H = 2·56 in. κ = Same	2·86 in. as for K ₁ .	3·35 in.
O	H = 6·95 in. κ = 356°	7·94 in. 5°	8·68 in. 358°

In these results there appears to be some evidence of a progressive change as the season advances, such as was noted in the case of the observations made by the "Discovery" in 1902-3†, and I shall return later to this subject. But in the case of the tides S₂, K₂, K₁, P, this might easily arise from an erroneous assumption as to the heights and phases of K₂ and P relatively to those of S₂ and K₁ respectively. It is therefore advisable to discuss these tides without making the assumptions which are necessary when each month is treated quite independently of the others.

* 'Roy. Soc. Proc.' 1892, vol. 52, p. 345, or 'Scientific Papers,' vol. 1, Paper 6.

† 'National Antarctic Expedition, 1901-4, Physical Observations' (1908), p. 3; or my 'Scientific Papers,' vol. 1 (1907), Paper 12.

In explaining my procedure, I adopt the notation of my paper "On an Apparatus for Facilitating the Reduction of Tidal Observations."*

The heights and phases of the tides S_2 , K_2 , K_1 , P are denoted respectively by H_s, κ_s ; H'', κ'' ; H', κ' ; H_p, κ_p .

The pair of harmonic constituents for diurnal tides, when 27 consecutive days are analysed, are denoted by $\mathfrak{A}_1, \mathfrak{B}_1$, and the theory shows that

$$\left. \begin{matrix} \mathfrak{A}_1 \\ \mathfrak{B}_1 \end{matrix} \right\} = \frac{f'H'}{\mathfrak{F}_1} \cos(\kappa' - V' - 13^\circ.29) - \frac{H_p}{\mathfrak{F}_1} \cos(\kappa' - V' - 13^\circ.29 + 2h_0 - v' + 26^\circ.58 + \kappa_p - \kappa').$$

Similarly, when 30 consecutive days are analysed, and when P denotes the mean value for the month of the ratio of the cube of the sun's parallax to his mean parallax, the pair of semi-diurnal constituents are given by

$$\left. \begin{matrix} \mathfrak{A}_2 \\ \mathfrak{B}_2 \end{matrix} \right\} = PH_s \frac{\cos \kappa_s}{\mathfrak{F}_2} + \frac{f''H''}{\mathfrak{F}_2} \cos(\kappa_s - 2h_0 + 2v'' - 29^\circ.53 + \kappa'' - \kappa_s).$$

In treating each single month independently, we assumed $\kappa_p = \kappa'$, $\kappa_s = \kappa''$, $\frac{H_p}{H'} = \frac{1}{3}$, $\frac{H'}{H_s} = \frac{3}{11}$, but we now no longer make that supposition.

If we put

$$\left. \begin{matrix} a' \\ b' \end{matrix} \right\} = \frac{f'}{\mathfrak{F}_1} \cos(V' + 13^\circ.29); \quad \left. \begin{matrix} a_p \\ b_p \end{matrix} \right\} = \frac{1}{\mathfrak{F}_1} \cos(V_p - 13^\circ.29);$$

$$\left. \begin{matrix} A' \\ B' \end{matrix} \right\} = H' \frac{\cos \kappa'}{\sin \kappa'}; \quad \left. \begin{matrix} A_p \\ B_p \end{matrix} \right\} = H_p \frac{\cos \kappa_p}{\sin \kappa_p};$$

a' , b' , a_p , b_p , are known functions, and each month gives the pair of equations—

$$\mathfrak{A}_1 = a'A' + b'B' + a_pA_p + b_pB_p,$$

$$\mathfrak{B}_1 = -b'A' + a'B' - b_pA_p + a_pB_p.$$

Thus the three months afford six equations for the determination of A' , B' , A_p , B_p , from which the heights and phases of K_1 and P are easily found.

Again, if we put

$$\frac{a_s}{b_s} = P; \quad \left. \begin{matrix} a'' \\ b'' \end{matrix} \right\} = \frac{f''}{\mathfrak{F}_2} \cos(V'' + 29^\circ.53);$$

$$\left. \begin{matrix} A_s \\ B_s \end{matrix} \right\} = H_s \frac{\cos \kappa_s}{\sin \kappa_s}; \quad \left. \begin{matrix} A'' \\ B'' \end{matrix} \right\} = H'' \frac{\cos \kappa''}{\sin \kappa''};$$

each month gives for the semi-diurnal tides the pair of equations—

$$\mathfrak{A}_2 = a_sA_s + b_sB_s + a''A'' + b''B'',$$

$$\mathfrak{B}_2 = -b_sA_s + a_sB_s - b''A'' + a''B'';$$

* 'Roy. Soc. Proc.' 1892, vol. 52, pp. 345—389; or 'Scientific Papers,' vol. 1 (1907), Paper 6.

and the three months give six equations for determining A_s, B_s, A'', B'' , from which the heights and phases of S_2 and K_2 are easily found:

On solving the diurnal group of equations by least squares, I find $H' = 8.311$ inches, $\kappa' = 11^\circ 50'$, $H_p = 1.795$ inch, $\kappa_p = 12^\circ 11'$. The ratio of H' to H_p is 4.63, instead of the 3 assumed from theoretical considerations in the separate treatment of the months, but the phases are virtually identical. The similar treatment of the semi-diurnal group gives

$$H_s = 0.938 \text{ inch, } \kappa_s = 273^\circ 25'; H'' = 0.584 \text{ inch, } \kappa'' = 257^\circ 35'.$$

The ratio of H_s to H'' is 1.605, instead of $3\frac{2}{3}$, as assumed from theory.

It thus appears that the theoretical hypotheses were considerably in error, and results probably more in accordance with the truth will be obtained from the several months if we assume

$$H' = 4.63 H_p, \kappa_p - \kappa' = 0^\circ 22'; H_s = 1.605 H'', \kappa_s - \kappa'' = 15^\circ 50'.$$

With these assumptions the three months now give—

	1.	2.	3.
K_1	$H = 8.20 \text{ in.}$ $\kappa = 10^\circ$	8.31 in. 16°	8.58 in. 10°
P	$H = 1.77 \text{ in.}$ $\kappa = 11^\circ$	1.79 in. 17°	1.85 in. 11°
S_2	$H = 0.96 \text{ in.}$ $\kappa = 272^\circ$	0.93 in. 276°	0.94 in. 272°
K_2	$H = 0.60 \text{ in.}$ $\kappa = 256^\circ$	0.58 in. 260°	0.59 in. 256°

The appearance of progressive seasonal change in this group of tides has now almost disappeared, although the middle month is slightly discordant from the other two.

It is interesting to note that, in the result of the treatment by least squares, κ' (for K_1) is practically identical with κ_p (for P), but that there is a considerable divergence between κ_s (for S_2) and κ'' (for K_2).

The difference between the phase of M_2 (which we may take as given by $\kappa_m = 5^\circ$) and that of S_2 given by $\kappa_s = 273^\circ$ is very large, although their difference of speeds is not great. Hence we should expect that a small difference of speed in a semi-diurnal tide would make a sensible difference in phase.

If phase varies simply as difference of speed, we shall have the following results:—

$$\text{Speed of } S_2 - \text{speed of } M_2 = 1^\circ.016 \text{ per hour; } \kappa_s - \kappa_m = 273^\circ - 365^\circ = -92^\circ.$$

$$\text{Speed of } K_2 - \text{speed of } S_2 = 0^\circ.082 \text{ per hour.}$$

Hence we ought to find

$$\frac{\kappa'' - \kappa_s}{\kappa_s - \kappa_m} = \frac{0.082}{1.016}, \quad \text{or} \quad \kappa'' = \kappa_s - \frac{82}{1016} \times 92^\circ = \kappa_s - 7^\circ.$$

As a fact, we find $\kappa'' = \kappa_s - 16^\circ$, and thus the direction of the difference of phases is such as was to be expected, although the amount is not quite satisfactory. With tides of such small amplitude, however, and with only three months on which to rely, the amount of agreement is all that is to be expected.

The results of the analysis for the tides M_2 and O , when the months are taken independently, are given above. If, however, we neglect the equation of time, the whole period of three months may be treated as a single group of observations. In this way I obtain for M_2

$$\begin{aligned} H_m &= 2.4233 \text{ inches,} \\ \kappa_m &= 5^\circ 33'. \end{aligned}$$

If we take the three values of each of the quantities $H_m \cos \kappa_m$, $H_m \sin \kappa_m$, from our previous results, and form means of these functions, we obtain

$$\begin{aligned} H_m &= 2.4183 \text{ inches,} \\ \kappa_m &= 4^\circ 20'. \end{aligned}$$

The latter method has the advantage that it takes the equation of time into account; the former is somewhat more likely to eliminate casual inequalities. We may safely take $H_m = 2.42$ inches, $\kappa_m = 5^\circ$, as being very near the truth.

Similarly the whole series when treated for the O tide gives

$$\begin{aligned} H_o &= 8.1645 \text{ inches,} \\ \kappa_o &= 1^\circ 0'. \end{aligned}$$

But the means of the three values of $H_o \cos \kappa_o$, $H_o \sin \kappa_o$, give the somewhat discordant result

$$\begin{aligned} H_o &= 7.8407 \text{ inches,} \\ \kappa_o &= 359^\circ 45'. \end{aligned}$$

I should have expected the two evaluations to be closer together, as was the case with M_2 , and I think we must accept

$$\begin{aligned} H_o &= 8.0 \text{ inches,} \\ \kappa_o &= 0^\circ, \end{aligned}$$

as being as nearly accurate as is possible from our data.

In the reduction of the "Discovery" observations it was known that there had frequently been a small change of the zero point in consequence of the shift in the ship, and I did not think it was worth while to attempt

to combine the several months by least squares so as to separate the tides S_2 from K_2 , and K_1 from P . I now think that it was a pity that the attempt was not made to separate them, and therefore I have gone back to the old work and discussed the numbers by least squares with the results given below. In the course of this revision it appeared that there had been a small mistake in the value assigned to P for each month, which, however, made little change in the values assigned to the tide S_2 , and did nothing to remove the considerable discrepancies between the results from each of the 12 months.

FINAL TABLE OF RESULTS FOR "NIMROD," TOGETHER WITH COMPARISON WITH "DISCOVERY."

	"Nimrod," 1908.	"Discovery," 1902—3.	"Discovery," new reduction.
M_2	H = 2.42 in. = 0.202 ft. $\kappa = 5^\circ$	1.966 in. = 0.164 ft. 10°	
S_2	H = 0.94 in. = 0.078 ft. $\kappa = 273^\circ$	1.142 in. = 0.095 ft. 272°	1.129 in. = 0.094 ft. 272°
K_2	H = 0.584 in. = 0.049 ft. $\kappa = 258^\circ$	0.311 in. = 0.024 ft. 272°	0.396 in. = 0.033 ft. 294°
K_1	H = 8.31 in. = 0.693 ft. $\kappa = 12^\circ$	9.245 in. = 0.770 ft. 14°	10.177 in. = 0.848 ft. 14°
P	H = 1.795 in. = 0.150 ft. $\kappa = 12^\circ$	3.082 in. = 0.257 ft. 14°	3.228 in. = 0.269 ft. 3°
O	H = 8.0 in. = 0.67 ft. $\kappa = 0^\circ$	9.264 in. = 0.772 ft. 0°	

The agreement between these two sets of constants, deduced from observations taken at places some 25 miles apart, seems to be very good. The later observations were taken further north than the earlier ones, and the greater value of M_2 in the more northerly series is probably a reality. The two days of observation made by Dr. Wilson in 1904 close to the "Nimrod" station agree with our present results in indicating a slightly increased value of the semi-diurnal tide.

In discussing the "Discovery" tides, I was led to suspect that there were semi-diurnal nodal lines to the northward, but that the node for S_2 was nearer than that for M_2 . The fall in the amplitude of S_2 agrees with this, and possibly the amplitude of M_2 has begun to increase as we go northward previously to its subsequent decrease to the zero value at the node.

The ratio of M_2 to S_2 for "Nimrod" is 2.57, and for "Discovery" 1.74; the former value is more nearly normal than the latter.

The sums of the heights of M_2 , S_2 , K_2 , are respectively 3.94 inches for "Nimrod" and 3.49 for "Discovery."

The sums for K_1 , P , O are 18.1 inches for "Nimrod" and 22.7 inches for "Discovery." Thus for "Nimrod" the greatest diurnal tides are 4.6 times as great as the greatest semi-diurnal tides, while for "Discovery" the greatest diurnal tides are 6.5 times as great as the greatest semi-diurnal tides. This again emphasises the diminishing importance of the semi-diurnal tides as we penetrate to the south.

It should be remarked that the difference of phase of K_2 from that of S_2 in the new reduction for "Discovery" is not in accordance with the theoretical considerations adduced in support of the corresponding difference for "Nimrod." However, too much stress should not be placed on results derived from these very small tidal oscillations.

On the whole, I conclude that we now know the tidal constants at this part of the Antarctic Ocean with as much accuracy as is desirable, and I refer the reader to the discussion of the "Discovery" observations for the conclusions which may be drawn from the values found.

In discussing the "Discovery" observations, I saw reason to suspect a remarkable seasonal change in the amplitude and phase of the tide M_2 ; it is therefore interesting to see whether these new observations tend to confirm that conclusion. The results in my previous paper were discussed by means of curves, but I will now merely examine the matter numerically.

The results for each month which has been reduced, viz., 12 for "Discovery" and 3 for "Nimrod," may be held to appertain to the middle of the month under consideration, that is to say 15 days after the corresponding epoch.

The following table exhibits the values of H_m and κ_m for each of the

Date.	H_m .	κ_m .
	inches.	°
Apr. 21, 1903.....	1.91	-12
May 24, 1903.....	2.20	-5
" 27, 1902.....	2.27	1
June 20, 1902.....	2.29	2
" 30, 1903.....	2.33	4
July 29, 1903.....	2.41	9
* " 29, 1908.....	2.55 (2.08)	-5
Aug. 8, 1902.....	2.18	14
* " 28, 1908.....	2.64 (2.15)	11
" 29, 1903.....	2.18	15
Sept. 7, 1902.....	1.93	22
* " 27, 1903.....	2.11 (1.72)	5
Oct. 8, 1902.....	1.74	26
" 28, 1902.....	1.56	32
Nov. 28, 1902.....	1.21	31

15 months, together with the dates to which they may be held to apply. The new results are marked with an asterisk.

In order to judge of the progression in the heights, we should note that the mean H_m for the northern place is 2.42, and for the southern is 1.97. Hence we ought, perhaps, to reduce the three heights for "Nimrod," viz., 2.55, 2.64, 2.11, in the proportion of 197 to 242. The corresponding numbers as so reduced are written in parentheses after the actual numbers. The progression in the heights appears to be fairly consistent, but that of the phases is not nearly so clear. The phase of the first of the "Nimrod" months is some 15° away from what we should expect if the progressive change is an actuality. If we convert this into time, it means that the high water should be changed by about half an hour to fit into the supposed progression. The middle month fits into its place fairly well, but the high water for the third month should be shifted some 40 minutes. Such changes are not, however, large, when we consider that the range from high to low water is only about 5 inches. On the whole, I should say that the new results do not tend to confirm the truth of the progressive change in any marked degree, but they can hardly be held to invalidate it.

If we examine the results of the three months for the O tides, we find some traces of a seasonal progression, for the heights are 6.95, 7.94, 8.64; but the progression of phases is again not clearly marked, for they are -5° , $+5^\circ$, -2° . I was not able to detect any evidence of progression in the case of the O tide as observed by the "Discovery."

III.—ON SEA-SEICHES IN THE ANTARCTIC OCEAN.

In the course of the reduction of the tidal observations the mean daily heights of the water were computed, so as to furnish a cross verification of the summations necessary in the harmonic analysis. In view of the arduous conditions under which the observations were made it also seemed well to test the series of means, so as to detect any accidental shift in the zero of the gauge which might have occurred. Unfortunately, no such systematic examination of the "Discovery" observations had been carried out, because it was well known that there had been frequent small changes of zero due to the shift of the ship. It did not occur to me that a graphical illustration of mean sea-levels, known to be subject to somewhat frequent changes of zero, might give indications of anything worthy of notice.

A cursory examination of a table of the daily mean sea-levels of the present series at once revealed considerable inequalities. The paper on the drum was changed once a week, yet there was no sign of any weekly discontinuity, and the observers did not think there was any reason to suspect a change between

each paper and the next. A zigzag of daily mean sea-levels was accordingly plotted, as shown on a reduced scale in the firm line of fig. 4. I was surprised to see a somewhat regular rise and fall of the water with a period of about three days, for nearly five weeks on end. Although the rise and fall was then interrupted, this seemed to be a fact worth looking into.

A line drawn so as to bisect the zigzags clearly undergoes changes of considerable amount, for which it is only possible to guess the causes. Distant barometric changes and distant gales may be responsible for most of the effect. There are also probably annual and semi-annual meteorological tides, fortnightly and monthly astronomical tides, and some small apparent inequality with a period of a fortnight due to the residual effects of the tides of short period. But these causes obviously could not produce the shorter zigzags, so that we may consider these as being embroidered, to use M. Forel's phrase, on a slowly variable curve.

Local barometric changes must affect the mean sea-level, and pressure above the mean will correspond with depressed sea-level, at the rate of about $13\frac{1}{2}$ inches of water to one of mercury, and *vice versa*. Mr. James Murray has given me the mean barometric heights both in a tabular and in a graphical form. The means of pressure are given in civil time, while those of sea-level were computed according to astronomical time. I therefore made a rough estimate from the curve of the mean pressure according to the latter time. The mean pressure for the 90 days of observation was then found, and a correction was applied to the sea-levels at the rate of 14 inches of water to one of mercury above or below the estimated mean. A rather high value for the correction is taken, because it seemed desirable to give the barometric changes every possible chance of annulling the sea changes; and further, because, by the use of the factor 19 (instead of 18.857), the zigzags had been very slightly exaggerated. In any case, the correction is quite exact enough for such a rough allowance for barometric pressure as is possible.

The corrected mean sea-levels are shown in the dotted curve of fig. 4. It will be seen that the zigzags are sensibly diminished, but not annulled, and that in one or two places a new maximum or minimum has been introduced. We may conjecture that distant barometric changes and distant gales may have annulled some maxima and minima which would otherwise have been visible.

For observations of this uncertain kind mathematical treatment for the detection of partially veiled periodicity seems inappropriate. I have therefore only examined the zigzag for maxima and minima and have noted these incidences in the following table. In five cases a mark of *quære* is

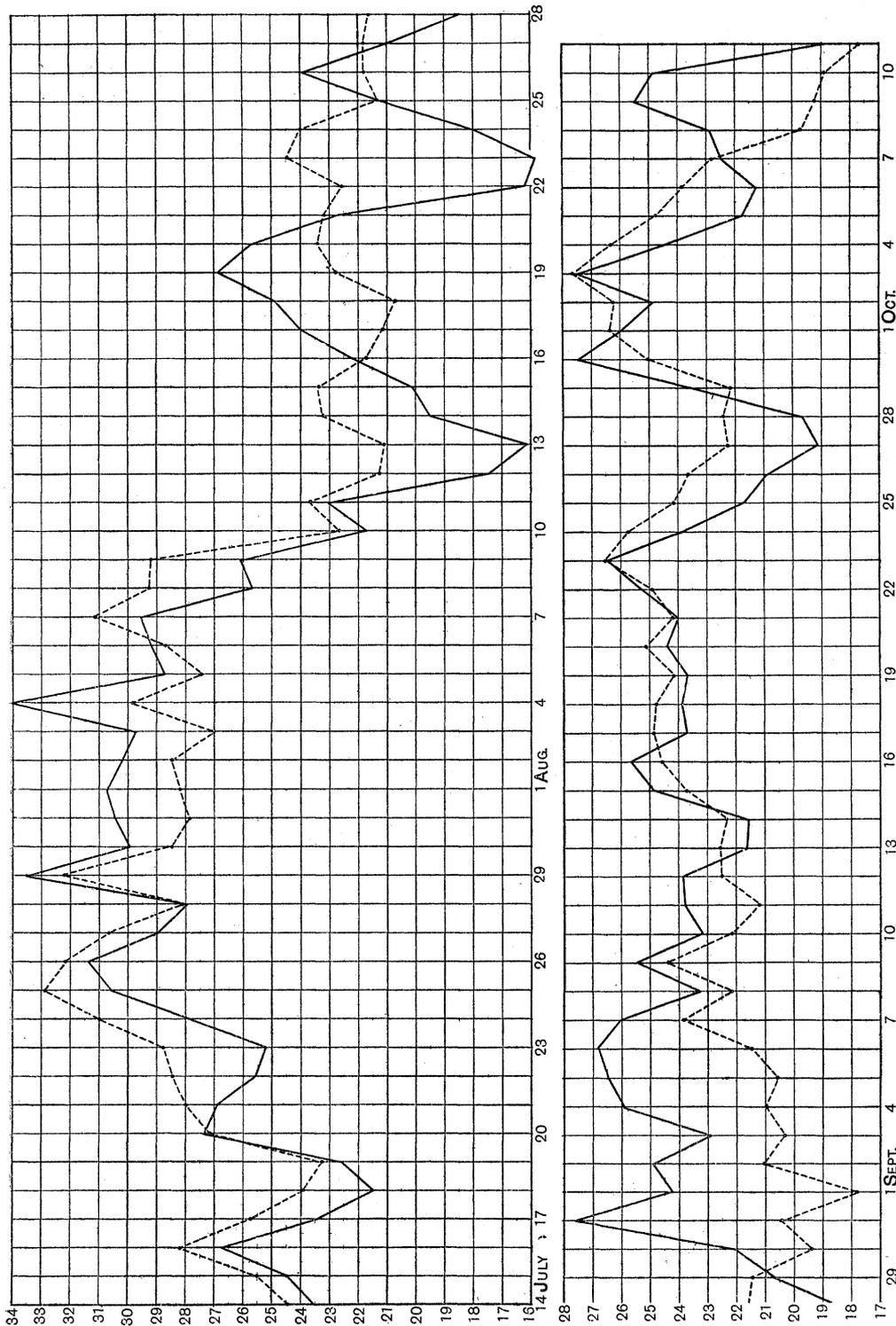


FIG. 4.—Daily means of sea-level, and the same corrected for barometric pressure, referred to an arbitrary zero and expressed in inches.

added because another observer might deny the existence of a maximum or minimum which I conceived to be there, but partially masked by the general rise or fall of an ideal line bisecting the zigzags of the dotted line. In the second column of each half of the table I give the differences between the dates in days, and these numbers will give the period of the suspected inequality.

TABLE OF MAXIMA AND MINIMA OF MEAN SEA-LEVEL CORRECTED FOR BAROMETRIC PRESSURE, 1908.

Maxima.	Periods in days.	Minima.	Periods in days.
July 16	5	July 14	5
21	4	19	4
25	4	23	5
29	4	28	3
Aug. 2	2	31	3
4	3	Aug. 3	2
7	2	5	2
9	2	7	3
11	$3\frac{1}{2}$	10	3
$14\frac{1}{2}$	$5\frac{1}{2}$	13	5
20	$3\frac{1}{2}$	18	4
$23\frac{3}{4}$	$3\frac{1}{2}$	22	3
27	4	25	3
31	2	28 ?	2
Sept. 2	2	30	2
4	3	Sept. 1	2
7	2	3	2
9	$3\frac{1}{2}$	5	3
$12\frac{1}{2}$	$4\frac{1}{2}$	8	3
17	3	11	3
20	3	14	5
23	3	19	2
26 ?	2	21	$3\frac{1}{2}$
28	3	$24\frac{1}{2}$?	$2\frac{1}{2}$
Oct. 1	2	27	2
3	4	29	3
7 ?	3	Oct. 2	3
10		5 ?	3
		8	3
		11	3

The intervals between successive maxima are as follows:—

$$1 \text{ of } 5\frac{1}{2} = 5\frac{1}{2}$$

$$1 \text{ of } 5 = 5$$

$$1 \text{ of } 4\frac{1}{2} = 4\frac{1}{2}$$

$$5 \text{ of } 4 = 20$$

$$4 \text{ of } 3\frac{1}{2} = 14$$

$$7 \text{ of } 3 = 21$$

$$8 \text{ of } 2 = 16$$

$$\text{Total of 27 periods} = 86 \text{ days.}$$

$$\text{Mean period} = 3.185 \text{ days.}$$

If we suppose the intervals of $5\frac{1}{2}$, 5, and $4\frac{1}{2}$ were really double periods, with masked maxima intervening, there were 30 periods, and the mean becomes 2.867 days.

The intervals between the successive minima are as follows:—

4 of 5	= 20	
2 of 4	= 8	
1 of $3\frac{1}{2}$	= $3\frac{1}{2}$	Total of 29 periods = 89 days.
13 of 3	= 39	Mean period = 3.069 days.
1 of $2\frac{1}{2}$	= $2\frac{1}{2}$	
8 of 2	= 16	

If the intervals of five days were really double periods with masked minima intervening, there were 33 periods and the mean period becomes 2.697 days.

Taking both estimates as of equal weight we get a mean interval of 3.127 days, or allowing for possible masked maxima or minima, as explained above, of 2.782 days. I think then that there is some evidence of the existence of an oscillation with a period of about three days.

In a paper in the 'Philosophical Magazine' (January, 1908, p. 88), Messrs. Honda, Terada, and Isitani discuss "Secondary Undulations of Oceanic Tides" or sea-seiches. These seiches occur in bays, and they find that the period depends on the size and depth of the bay. In some bays the period is fairly constant, but in others it changes "continuously and through certain ranges." They show that for a bay of length l and depth h the main period T is given by the formula $4l \div \sqrt{gh}$, where g is gravity. The period as so computed is subject to a correction due to the opening into the sea, but as we only now want a very rough estimate of the period, the correction may be neglected. The formula is the same as that for the period of the uninodal seiche in a lake of length $2l$. The authors in fact regard the end of the bay as resembling the end of a lake, while the seaward opening is equivalent to the middle of the lake. Accordingly the second half of the lake, which would stretch out into the sea, is suppressed. The formula gives results in accordance with the seiches observed in many Japanese bays, and they remark that bays are also sometimes disturbed by seiches of shorter period, which they regard as transverse seiches from side to side of the bay, just as if it were an enclosed basin.

In none of the examples given by these authors has the seiche a period at all comparable with that of which we have reason to suspect the existence in the Antarctic Sea, but that affords no reason for refraining to apply the theory to such prolonged oscillations. In most inland lakes the seiches have periods of 10 minutes to one or two hours, yet in Lake Erie the seiche is found to have a period of 13 hours, while in the Lakes of

Michigan and Huron conjointly a seiche of 45 hours is suspected.* Thus we have justification for the application of the theory to oscillations of very long period.

In the case of the Antarctic Sea, if there is a great bay running far back into the Antarctic continent behind the ice barrier, its length and depth are quite unknown. Hence there are elements of great uncertainty in the application of the theory.

It seems likely, at any rate, that the bay extends for a considerable distance, and speculations have even been made as to whether there may not be an arm of the sea stretching through to the Weddell Sea almost diametrically across what was supposed to be a continent.

It might, perhaps, be thought that the thick ice of the barrier would serve to damp out oscillations of sea-level; but, unless, indeed, the sea is solid to the bottom, I conceive that the ice would behave like an elastic skin, and would hardly exercise any damping effect on oscillations with a period of more than an hour or two.

It seems almost impossible that the remarkable changes of sea-level which are observed should arise from errors of observation, and if they exist at Backdoor Bay, the neighbouring sea along the barrier must necessarily also partake of the motion. If the sea rises and falls, the barrier itself, must move with it; and it may be suspected that it is subject to a true tidal rise and fall.

If we accept the existence of a sea-seiche with a period of three days, the formula gives some indication as to the length and depth of the bay behind the barrier. We cannot assume the sea to be very shallow, because if it were so it would inevitably be frozen solid to the bottom. Moreover, a shallow sea would certainly be broken up by shoals, so that it could not oscillate as a single system. A little consideration shows that to produce a seiche of three days the bay must be of enormous length, and for the reasons assigned it would be useless to assume it to be very shallow. The few soundings near the barrier give depths of between 200 and 300 fathoms, and perhaps a somewhat smaller depth might suffice to allow of the required seiche. I propose to guess the length of the bay and to find what depth of sea is required to produce a seiche of three-day period.

I guess then that the bay behind the barrier stretches past the South Pole and a little to the east of it as far as latitude 80° . Such an inlet would have a length of 25° to 30° of latitude. It seems likely that if it is really an arm of the sea through to Weddell's Sea, with a constriction about the place where we place the end of the bay, the seiche would be much the same.

* Dr. Anton Endrös, 'Petermann's Geograph. Mitteilungen,' Heft II, 1908.

The length of our supposed bay in centimetres will be 25 or 30 times $60 \times 1.852 \times 10^5$ cm., and these I take as two assumed values of l . On completing the multiplications I find that $4l$ will be $1\frac{1}{3} \times 10^9$ cm. or $1\frac{1}{3} \times 10^9$ cm.

The period of oscillation is three days, or 2.592×10^5 sec.; also g is 981. Thus, numbering our two alternatives as (1) and (2), we get:—

$$(1) \ 2.592 \times 10^5 = \frac{1\frac{1}{3} \times 10^9}{\sqrt{981h}}; \quad (2) \ 2.592 \times 10^5 = \frac{1\frac{1}{3} \times 10^9}{\sqrt{981h}}.$$

Whence

$$\begin{aligned} (1) \ h &= \frac{1}{981} \left(\frac{1\frac{1}{3} \times 10^4}{2.592} \right)^2 & (2) \ h &= \frac{1}{981} \left(\frac{1\frac{1}{3} \times 10^4}{2.592} \right)^2 \\ &= 18,732 \text{ cm.} & &= 26,975 \text{ cm.} \\ &= 102.4 \text{ fathoms.} & &= 147.5 \text{ fathoms.} \end{aligned}$$

Thus a sea of from 100 to 150 fathoms in such an immense bay as has been conjectured would oscillate with a period of three days, and the observed results are seen to be consistent with the existence of a deep inlet, almost or quite cutting the Antarctic continent in two.

Such a conclusion is interesting, but it would not be right to attribute to it a high degree of probability, because there are elements of uncertainty on every side.

In view of the interest of our result it has seemed well to revert to the observations made by Captain Scott's expedition, notwithstanding the known uncertainty in the zero of the gauge. I have therefore examined 175 days of the "Discovery's" record, viz., 113 days of 1902 and 62 days of 1903. No correction has been applied for barometric pressure, and thus periodic inequalities have doubtless sometimes been masked by contemporaneous changes of pressure, and perhaps by the shift of zero. Thus I should expect to find rather a larger proportion of long intervals between consecutive maxima and minima than in the "Nimrod" results as reduced for pressure. I found, in fact, on analysing the zigzag in the way already explained that, amongst the periods as deduced from maxima, there were:—

2 of 7 d., 1 of $6\frac{1}{2}$ d., 10 of 5 d., 1 of $4\frac{1}{2}$ d.,

and amongst the periods, as deduced from minima, there were—

1 of 9 d., 1 of $6\frac{1}{2}$ d., 4 of 6 d., 3 of 5 d., 1 of $4\frac{1}{2}$ d.

Taking maxima and minima together there were 83 periods amounting to 323 days, thus giving a mean period of 3.9 days.

But if we postulate that periods from 7 days to $4\frac{1}{2}$ days were really double periods with masked maxima or minima intervening, and that the

9-day period is really triple, we get 105 periods for 323 days, with a mean of 3.1 days.

These results are generally confirmatory of the preceding ones, but seem to indicate a slightly longer period.

In this rough examination there is undoubtedly a danger of finding a false periodicity under the influence of unconscious bias. I thought it advisable therefore to examine other tidal records, for it might be possible to perceive periodicity even in cases where there was but small likelihood of its real existence. Colonel Burrard then kindly sent me tables of daily mean sea-levels for the year 1880 from May 1 to June 30 and from October 1 to November 30 for Aden, Karachi, Madras and Port Blair, Andaman Islands. These old observations were chosen because it had been usual at that time to have each daily mean "cleared" of the residual effects of the tides of short period, and thus one slight source of error was obviated. I also proceeded in the case of Aden to deduct the tides of long period, but as this correction clearly made no difference in the kind of inequality I was looking for, I did not carry out that laborious task in the other cases.

The tabulated numbers were then plotted out in a number of curves.

An imaginative investigator might possibly fancy he could detect signs of periodicity with a period of two or three days at Aden and at Port Blair, but as the range from crest to hollow was not more than $\frac{1}{2}$ inch, it seems safer to say that no periodicity could be traced.

In the curve for Madras there are considerable irregularities, but it seemed impossible even to imagine any periodicity. At Karachi there does seem to be an inequality with a period of two to three days and a range of two or three inches. A succession of waves with three to five crests one after the other is observable at several parts of the curve. It seems quite likely that sea-seiches may exist in the Indian Ocean, and Karachi would be well placed for observing them.

These Indian results were not corrected for barometric pressure, and it may be worth while hereafter to submit them to a more systematic examination. For the present, however, I am satisfied with the conclusion that periodicity is not to be seen in all cases, and that the oscillations of mean sea-level in the Antarctic Sea are many times as great as those in the Indian Ocean and Bay of Bengal. Thus it seems unlikely that imagination is responsible for the existence of the Antarctic sea-seiches, and we may hope that the investigations of Captain Scott's second expedition will throw some further light on the subject, and possibly also on the existence of a deep bay behind the barrier.
