nerating line is perpendicular to one of the coordinate planes, the line drawn from the centre to the point where this line intersects the surface is a conjugate diameter to this plane, a result which might be obtained from geometrical considerations.

We may, as a simple consequence from the preceding demonstration, obtain a theorem in spherical trigonometry apparently new.

Let a, b, c be the sides of a spherical triangle, and P'', P', P *the arcs of three great circles drawn from the vertices* A, B, C *of the spherical triangle, through a point* S *assumed on the surface of the sphere to the opposite sides,* p'' p' p *the segments of those arcs between the point* S *and the sides a, b, c,* we shall have

$$\left[\frac{\sin p}{\sin P} + \frac{\sin p'}{\sin P'} + \frac{\sin p''}{\sin P''}\right]^2 = 1$$

$$+ 2\frac{\sin p \sin p' \sin p''}{\sin P \sin P' \sin P''}\left[\frac{\sin P}{\sin p}(1 - \cos c) + \frac{\sin P'}{\sin p'}(1 - \cos b) + \frac{\sin P''}{\sin p''}(1 - \cos a)\right]$$

To show this, through the point O let a right line be drawn parallel to P Q, meeting the surface of the sphere in S, and let the sides of the spherical triangle, opposite the angles λ, μ, ν, be a, b, c; then in the triangle P C D : P D : P C :: sin P C D : sin P D C : sin p : sin P.

Since P D is parallel to O Z, and P C parallel to O S, hence $\frac{z}{c} = \frac{\sin p}{\sin P}$.

Similarly, $\frac{x}{a} = \frac{\sin p''}{\sin P''}$, $\frac{y}{b} = \frac{\sin p'}{\sin P'}$; making these substitutions in (2.), after some obvious simplifications we find

$$\left[\frac{\sin p}{\sin P} + \frac{\sin p'}{\sin P'} + \frac{\sin p''}{\sin P''}\right]^2 = 1$$

$$+ 2\frac{\sin p \sin p' \sin p''}{\sin P \sin P' \sin P''}\left[\frac{\sin P}{\sin p}(1 - \cos c) + \frac{\sin P'}{\sin p'}(1 - \cos b) + \frac{\sin P''}{\sin p''}(1 - \cos a)\right]$$

When the triangle becomes plane, the sines are changed into the corresponding arcs, and cos a, cos b, cos c are each equal to unity, and we thus derive the known theorem in plane geometry,

$$\frac{p}{P} + \frac{p'}{P'} + \frac{p''}{P''} = 1.$$

Phil. Mag. 3^d. XXI p. 180

XXX. *Notes on the Effects produced by the Ancient Glaciers of Caernarvonshire, and on the Boulders transported by Floating Ice.* By CHARLES DARWIN, *Esq., M.A., F.R.S. and F.G.S.*

GUIDED and taught by the abstract of Dr. Buckland's memoir "On Diluvio-Glacial Phænomena in Snowdonia and the adjacent parts of North Wales *," I visited several of the localities there noticed, and having familiarized myself with some of the appearances described, I have been enabled to make a few additional observations.

Dr. Buckland has stated that a mile east of Lake Ogwyn there occurs a series of mounds, covered with hundreds of large blocks of stone, which approach nearer to the condition of an undisturbed moraine, than any other mounds of detritus noticed by him in North Wales. By ascending these mounds it is indeed easy to imagine that they formed the north-western lateral moraine of a glacier, descending in a north-east line from the Great Glyder mountain. But at the southern end of Lake Idwell the phænomena of moraines are presented, though on a much smaller scale, with perfect distinctness. On entering the wild amphitheatre in which Lake Idwell lies, some small conical, irregular little mounds, which might easily escape attention, may be seen at the further end. The best preserved mounds lie on the west side of the great black perpendicular face of rock, forming the southern boundary of the lake. They have been intersected in many places by streams, and they are seen to consist of earth and detritus, with great blocks of rock on their summits. They at first appear quite irregularly grouped, but to a person ascending any one of those furthest from the precipice, they are at once seen to fall into three (with traces of a fourth) narrow straight linear ridges. The ridge nearest the precipice runs someway up the mountain, but the outer one is longer and more perfect, and forms a trough with the mountain-side, from 10 to 15 feet deep. On the eastern and opposite side of the head of the lake, corresponding but less developed mounds of detritus may be seen running a little way up the mountain. It is, I think, impossible for any one who has read the descriptions of the moraines bordering the existing glaciers in the Alps, to stand on these mounds and for an instant to doubt that they are ancient moraines; nor is it possible to conceive any other cause which could have abruptly thrown up these long narrow steep mounds of unstratified detritus against the mountain-sides. The three

* Read before the Geological Society, December 15th, 1841, and the Abstract is published in the Athenæum, 1842, p. 42. [An Abstract of Dr. Buckland's paper, from the Proceedings of the Society, will appear in an early number of the Philosophical Magazine.—EDIT.]

or four linear ridges evidently mark the principal stages in the retreat of the glacier; the outer one is the longest, and diverges most from the great wall of rock at the south end of the lake. The inner lines distinctly define the boundary of the glacier during the last stage of its existence. At this period a small and distinct glacier descended from a narrow but lofty gorge on the north-western end of the lake; and here remnants of a terminal moraine may be traced in the little mounds, forming a broken semicircle round a rushy plain, scarcely more than a hundred yards in diameter. The rocks are smoothed, mammillated and scored, all round the lake, and at some little depth beneath the surface of the water, as I could both see and feel. Similar marks occur at great heights on all sides, far above the limits of the moraines just described, and were produced at the time when the ice poured in a vast stream over the rocky barrier bounding the northern end of the amphitheatre of Lake Idwell.

I may here mention, that about eighty yards west of the spot where the river escapes from the lake, through a low mound of detritus, probably once a terminal moraine, there is an example of a boulder broken, as described by Charpentier and Agassiz, into pieces, from falling through a crevice in the ice. The boulder now consists of four great tabular masses, two of which rest on their edges, and two have partly fallen over against a neighbouring boulder. From the distance, though small in itself, at which the four pieces are separated from each other, they must have been pitched into their present position with great force; and as the two upright thin tabular pieces are placed transversely to the gentle slope on which they stand, it is scarcely possible to conceive that they could have been rolled down from the mountain behind them; one is led, therefore, to conclude that they were dropped nearly vertically from a height into their present places.

The rocky and steep barrier over which the ice from the amphitheatre of Lake Idwell flowed into the valley of Nant-Francon, presents from its summit to its very foot (between 400 and 500 feet) the most striking examples of boss or dome-formed rocks; so much so, that they might have served as models for some of the plates in Agassiz's work on Glaciers. When two of the bosses stand near and are separated only by a little gorge, their steep rounded sides are generally distinctly scored with lines, slightly dipping towards the great valley in front. The summit of the bosses is comparatively seldom scored; but on one close to the bridge over the river Ogwyn, I remarked some singular zigzag scores. At this spot the cleavage of the slate is highly inclined, and owing apparently to the different degrees of hardness of the laminæ, smooth and gentle furrows have been produced by the grinding of the ice, transversely to the scores, and to the probable course of the glacier. Here, as well as in some few other places, I noticed an appearance which made it vividly clear that these bosses had been formed by some process quite different from ordinary aqueous or atmospheric erosion; it is the abrupt projection from the smooth surface of a boss of a piece of rock a few yards square, and one or two feet in height, with its surface smoothed and scored like the boss on which it stands, but with its sides jagged: if a statuary were to cut a small figure out of a larger one, the abrupt projecting portions, before he quite completed his work, might be compared to these masses of rock: how it comes that the glacier, in grinding down a boss to a smaller size, should ever leave a small portion apparently untouched, I do not understand.

On the summit of some of the bosses on this barrier there are perched boulders: but this phænomenon is seen far more strikingly close to Capel-Curig, where almost every dome of rock south of the Inn is surmounted by one or more large angular masses of foreign rock. The contrast between the rude form of these blocks, and the smooth mammillated domes on which they rest, struck me as one of the most remarkable effects produced by the passage of the glaciers. On the sides of the mountains above Capel-Curig, I observed some boulders left sticking on very narrow shelves of rocks, and other boulders of vast size scattered in groups. The largest boulder I noticed there was about 26 feet in length by 12 in breadth, and buried to an unknown thickness.

Proceeding down the great straight valley of Nant-Francon, which must formerly have conveyed the united glaciers from Lakes Idwell and Ogwyn, we continue to meet with boss-formed rocks till below the village of Bethesda. From this point towards Bangor these boss-formed rocks become rare; at least it is certain that a large number of hummocks of rock with rugged surfaces project, whereas higher up in this valley, and in all the great central valleys of Snowdonia, such unground hummocks are not to be met with. At Bethesda, unstratified masses of whitish earth, from ten to forty feet in thickness, full of boulders mostly rounded, but some angular, from one to four feet square, are first met with. This deposit is interesting from the boulders being deeply scored, like the rocks *in situ* over which a glacier has passed. The scores are sometimes irregular and crooked, but generally quite parallel, as I distinctly saw over the entire side of one large block. Some of the blocks were scored only on one side, others on two sides, but from the difficulty of turning over the larger ones, I do not know which case is most common. I saw one large block on which the scores on the opposite sides were all parallel; and another irregularly conical one, four feet in length, of which three-fourths of the circumference was marked with parallel striæ, converging towards the apex. In the smaller *elongated* blocks, from six to twelve inches in diameter, I observed that the striæ were generally, if not always, parallel to their longer axes, which shows that when subjected to the abrading force, they arranged themselves in lines of least resistance. Out of three large blocks which remained imbedded in a perpendicular cliff, the vertical sides of two were scored in horizontal lines, and of the third in an oblique direction. These several facts, especially the parallel striæ on the upper and lower surfaces, show that the boulders were not scored on the spot where they are now imbedded, as seems to have been the case with the boulders described by Mr Maclaren* in the till near Edinburgh. The contrast is very striking in the state of the surface of these boulders, and those which lie scattered high up on the sides of the adjoining hills and of the great central valleys, or are perched on the worn bosses of naked rock; such boulders, as I particularly noticed, present no signs of scores or striæ, as might have been anticipated, if, as is supposed, they were transported on the surface of the glaciers. In the quarries which I examined, namely, below Bethesda, and at some little height on the eastern side of the village, the till rested on slate-rocks, not worn into bosses. I found, however, a rather smooth pap of greenstone marked with a few deep scores. The till forms, at the height probably of 600 feet above the sea, a little plain, sloping seaward; and between Bethesda and Bangor, there are other gently inclined surfaces composed of till and stratified gravel. Considering these facts, together with the proofs of recent elevation of this coast, hereafter to be mentioned, I cannot doubt that this till was accumulated in a sloping sheet beneath the waters of the sea. In composition it resembles some of the beds of till in Tierra del Fuego, which have undoubtedly had this origin. I presume the scored, rounded, and striated boulders were pushed, in the form of a terminal moraine, into the sea, by the great glacier which descended Nant-Francon.

Mr. Trimmer† reports, on the *authority of some workmen*,

* Geology of Fife and the Lothians, p. 212.

† Proceedings of the Geological Society, vol. i. p. 332, or Phil. Mag. S. 2. vol. x. p. 143. Mr. Trimmer was one of the earliest observers of the scores and other marks on the rocks of North Wales. He has also remarked that "some of the larger blocks amid the gravel have deep scratches upon their surface." Mr. Trimmer himself found broken sea-shells in the diluvium at Beaumaris.

that sea-shells have been found on Moel Faban, two miles N.E. of Bethesda. I ascended this and some neighbouring hills, but could find no trace of any deposit likely to include shells. This hill stands isolated, out of the course of the glaciers from the central valleys; it exceeds 1000 feet in height; its surface is jagged, and presents not the smallest appearance of the passage of glaciers: but high up on its flanks (and perhaps on its very summit) there are large, angular and rounded boulders of foreign rocks.

Along the sea-coast between Bangor and Caernarvon, and on the Caernarvonshire plain, I did not notice any boss-formed hillocks of rock. The whole country is in most places concealed by beds of till and stratified gravel, with scattered boulders on the surface: some of these boulders were scored. From the account given by Mr. Trimmer* of his remarkable discovery of broken fragments of Buccinum, Venus, Natica, and Turbo, beneath twenty feet of sand and gravel, on Moel Tryfan (S.E. of Caernarvon), I ascended this hill. Its height is 1192 feet† above the sea; it is strewed with boulders of foreign rock, most of them apparently from the neighbouring mountains; but near the summit I found the rounded chalk-flints‡ and small pieces of white granite alluded to by Dr. Buckland. Its form is conical, and it stands isolated: wherever the bare rock protrudes its surface is jagged, and shows no signs of being in any part worn into bosses. The contrast between the superficial part of the bare rock on this hill and on Moel Faban, with that of the rocks within the great central valleys of Caernarvonshire, is very remarkable; it is a contrast of precisely the same kind as may be observed in these same valleys by ascending on either side above the reach of the ancient glaciers. A little way down the hill, a bed two or three feet in thickness, of broken fragments of slate mixed with a few imperfectly rounded pebbles and boulders of many kinds of rock, is seen in several places to rest on the slate, the upper surface of which, to the depth of several feet, has been disintegrated, shattered and contorted in a very curious manner. The laminated fragments, however, sometimes partially retain their original position.

I did not succeed in finding any fragments of shells, but near the summit of the hill on the eastern or inland side, I found beds, at least twenty feet in thickness, of irregularly stratified gravel and boulders, with distinct and quite defined layers of coarse yellow sand, and others of a fine argillaceous

* Proceedings of the Geological Society, vol. i. p. 332. [Phil. Mag. *loc. cit.*]

† Murchison's Silurian System, p. 528.

‡ I may mention, that at Little Madely, in Staffordshire, I have found chalk-flints in the gravel-beds, associated with existing species of sea-shells.

nature and reddish colour. These beds closely resemble those of Shropshire and Staffordshire, in which are found (as I have myself observed in very many places) fragments of sea-shells, and which every one, I believe, since the publication of Mr. Murchison's chapters on the drift of these counties, admits are of submarine origin. It may therefore be concluded that the layers of coarse and argillaceous sand, and of gravel, with far-transported pebbles and boulders, do not owe their origin to an inundation, but were deposited when the summit of Moel Tryfan stood submerged beneath the surface of the sea. As there are no marks of the passage of glaciers over this mountain (which indeed from its position could hardly have happened), we must suppose that the boulders were transported on floating ice; and this accords with the remote origin of some of the pebbles, and with the presence of the sea-shells. Within the central valleys of Snowdonia, the boulders appear to belong entirely to the rocks of the country. May we not conjecture that the icebergs, grating over the surface, and being lifted up and down by the tides, shattered and pounded the soft slate-rocks, in the same manner as they appear to have contorted the sedimentary beds of the east coast of England (as shown by Mr. Lyell)*, and of Tierra del Fuego? Although I was unable to find any beds on Moel Faban likely to preserve sea-shells, yet, considering the absence of the marks of the passage of glaciers over it, I cannot doubt that the boulders on its surface were transported on floating ice.

The drifting to and fro, and grounding of numerous icebergs during long periods near successive uprising coast-lines, the bottom being thus often stirred up and fragments of rock dropped on it, will account for the sloping plain of unstratified till, occasionally associated with beds of sand and gravel, which fringes to the west and north the great Caernarvonshire mountains.

In a paper read before the Geological Society †, I have remarked that blocks of rock are transported by floating ice under different conditions; 1st, by the freezing of the sea, in countries where the climate does not favour the low descent of glaciers; 2nd, by the formation of icebergs by the descent of glaciers into the sea, from mountains not very lofty, in latitudes (for instance in that of Geneva, or of the mouth of the Loire, in the southern hemisphere) where the surface of the

* "On the Boulder Formation of Eastern Norfolk;" Phil. Mag., S. 3, vol. xvi. May 1840, p. 351.

† May 5th. 1841, "On the distribution of the Erratic Boulders, and on the contemporaneous unstratified deposits of South America." [Phil. Mag. S. 3, vol. xix. p. 536.]

sea never freezes; and 3rd, by these two agencies united. I have further remarked that the condition and kind of the stones transported, would generally be influenced by the manner of production of the floating ice. In accordance with these views, I may remark that it does not seem probable from the low level of the Chalk-formation in Great Britain, that rounded chalk-flints could often have fallen on the surface of glaciers, even in the coldest times. I infer therefore that such pebbles were probably inclosed by the freezing of the water on the ancient sea-coasts. We have, however, the clearest proofs of the existence of glaciers in this country; and it appears, that when the land stood at a lower level, some of the glaciers, as in Nant-Francon, reached the sea, where icebergs charged with fragments would occasionally be formed. By this means we may suppose that the great *angular* blocks of Welch rocks, scattered over the central counties of England, were transported*. I looked carefully in the valleys near Capel-Curig and in Nant-Francon for beds of pebbles, or other marks of marine erosion, but could not discover any: when, however, Moel Tryfan and Faban stood beneath the level of the sea, inland creeks of salt-water must have stretched far up or quite through these valleys, and where they were deep, the glaciers (as at present in Spitzbergen†) would have extended, floating on the surface of the water, ready to become detached in large portions. From the presence of boss-formed rocks low down in the valley of Nant-Francon, and on the shores of the Lakes

* On the summit of Ashley Heath in Staffordshire, there is an angular block of syenitic greenstone, four feet and a half by four feet square, and two feet in thickness. This point is 803 feet above the level of the sea. From this fact, together with those relating to Moel Tryfan and Faban, we must, I think, conclude that the whole of this part of England was, at the period of the floating ice, deeply submerged. From the reasons given in my paper (Phil. Trans., 1839 [Phil. Mag. S. 3, vol. xiv. p. 363.]), I do not doubt that at this same period the central parts of Scotland stood at least 1300 feet beneath the present level, and that its emergence has since been very slow. The boulder on Ashley Heath probably has been exposed to atmospheric disintegration for a longer period than any other in this part of England. I was therefore interested in comparing the state of its lower surface, which was buried two feet deep in compact ferruginous sand (containing only quartz pebbles from the subjacent new red sandstone), with the upper part. I could not, however, perceive the smallest difference in the preservation of the sharp outlines of its sides. I had a hole dug under another large boulder of dark green felspathic slaty rock, lying at a lower level; it was separated by 18 inches of sand, (containing two pebbles of granite, and some angular and rounded masses of new red sandstone) from the surface of the new red sandstone. One of the rounded balls of this latter stone had been split into two, and deeply scored, evidently by the stranding of the boulder.

† Dr. Martens on the Glaciers of Spitzbergen, New Edinb. Phil. Journ. 1841, (vol. xxx.) p. 288.

of Llanberis (310 feet above the sea), it is evident that glaciers filled the valleys after the land had risen to nearly its present height; and these glaciers must have swept the valleys clean of all the rubbish left by the sea. As far as my very limited observations serve, I suspect that boss or dome-formed rocks will serve as one of the best criterions between the effects produced by the passage of glaciers and of icebergs*.

Dr. Buckland has described in detail the marks of the passage of glaciers along nearly the whole course of the great central Welch valleys; I observed that these marks were evident at the height of some hundred feet on the mountain-sides, above the water-sheds, where the streams flowing into the sea at Conway, Bangor, Caernarvon, and Tremadoc, divide: hence it appears that a person starting from any one of these four places (or from some way up the valley where the glacier ended), might formerly, without getting off the ice, have come out at either of the other three places, or low down in the valleys in which they stand. The mountains at this period must have formed islands, separated from each other by rivers of ice, and surrounded by the sea. The thickness of the ice in several of the valleys has been great. In the vale of Llanberis I ascended a very steep mountain, E.N.E. of the upper end of the upper lake, which slightly projects where the valley bends a little. For the lower 1000 feet (estimated, I think, correctly) the marks left by the glacier are very distinct, especially near the upper limit, where there are boulders perched on bosses of rock, and where the scores on the nearly vertical faces of rock are, I think, more distinct than any others which I saw. These scores are generally slightly inclined, but at various angles, seaward, as the surface of the glacier must formerly have been. But on one particular face of rock, inclined at an angle of somewhere about fifty degrees, continuous, well-marked and nearly parallel lines sloped upwards (in a contrary sense to the surface of the glacier) at an angle of 18° with the horizon. This face of rock did not lie parallel to the sides of the main valley, but formed one side of the sloping end of the mountain, over and round which the ice appears to have swept with prodigious force, expanding laterally after being closely confined by the shoulder above

* In the Appendix to my Journal of Researches (1839), I endeavoured to show that many of the appearances attributed to debacles, and to the movements of glaciers on solid land, would in all probability be produced by the action of stranded icebergs. I have stated (p. 619), on the authority of Dr. Richardson, that the rocky beds of the rivers in North America which convey ice, are smoothed and polished; and that (p. 620) the icebergs on the Arctic shore drive before them every pebble, and leave the submarine ledges of rock absolutely bare.

mentioned. At this point, where the glacier has swept to the westward, and has expanded, its surface seems in a short space to have declined much: for on a hill lying about a quarter of a mile N.W. of the shoulder, and forming a lower part of the same range (it stands S.S.E. of the Victoria Inn, and has a reddish summit), the marks of the passage of the glacier are at a considerably lower level. At the very summit, however, of this hill, several large blocks of rock have been moved from their places, as if the ice had occasionally passed over the summit, but not for periods long enough to have worn it smooth.

I cannot imagine a more instructive and interesting lesson for any one who wishes (as I did) to learn the effects produced by the passage of glaciers, than to ascend a mountain like one of those south of the upper lake of Llanberis, constituted of the same kind of rock and similarly stratified, from top to bottom. The lower portions consist entirely of convex domes or bosses of naked rock, generally smoothed, but with their steep faces often deeply scored in nearly horizontal lines, and with their summits occasionally crowned by perched boulders of foreign rock. The upper portions, on the other hand, are less naked, and the jagged ends of the slaty rocks project through the turf in irregular hummocks; no smooth bosses, no scored surfaces, no boulders are to be seen, and this change is effected by an ascent of only a few yards! So great is the contrast, that any one viewing these mountains from a distance, would in many cases naturally conclude that their bases and their summits were composed of quite different formations.

XXXI. *Application to particular instances of the general Formula for eliminating the Weights of Mixed Bases. By* JOSIAH REES, JUN., *F.G.S., of Her Majesty's Ordnance Geological Survey**.

THE general formula for eliminating the weights of any two bases, where the whole weight of any particular acid with which they are combined has been previously ascertained, is not easily available to those who are unaccustomed to mathematical inquiry.

If, however, we apply the general rule to particular instances, we are enabled to obtain a very simple place for each, by the application of which the weight of the bases may be ascertained.

I have thought it would not be altogether useless to draw up a few such rules for the use of chemists.

* Communicated by the Author.